Non–bipartite regular 2–factor isomorphic graphs: an update

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joint works with M. Abreu, M. Funk, B. Jackson, J. Sheehan et al.

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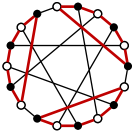
2–factors in regular graphs

- A 2–factor of G is a 2–regular spanning subgraph (i.e. it is a union of disjoint circuits that span G).



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- A 2–factor of G is a 2–regular spanning subgraph (i.e. it is a union of disjoint circuits that span G).

Problem (1)

Characterize regular graphs that possess only hamiltonian 2–factors *i.e.* 2–factor hamiltonian graphs*.*

Problem (2)

Characterize regular graphs with particular conditions on their 2–factors (e.g. (pseudo) 2–factor isomorphic graphs*).*

Problem (1): "2–factor hamiltonian graphs"

Definition

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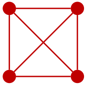
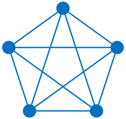


K_4



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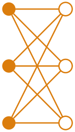
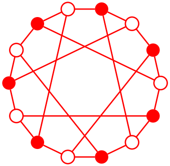


$K_{3,3}$



Heawood

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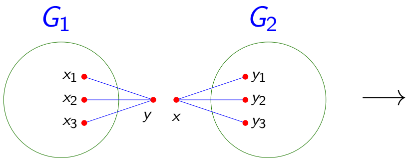

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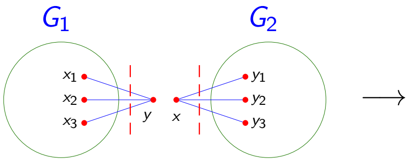

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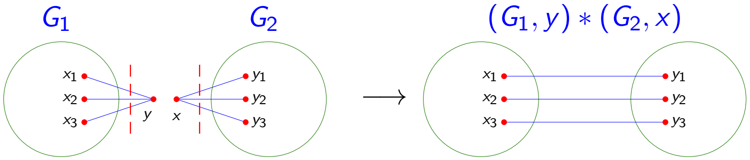

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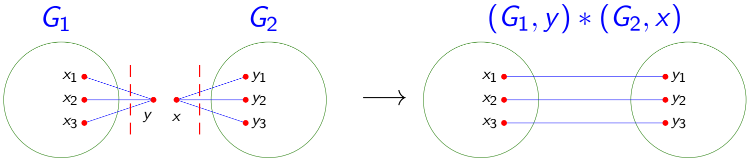

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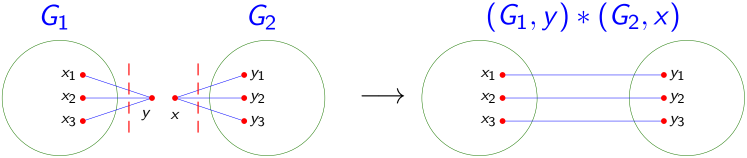

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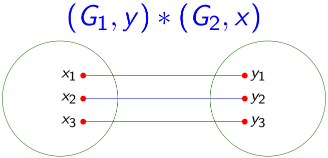

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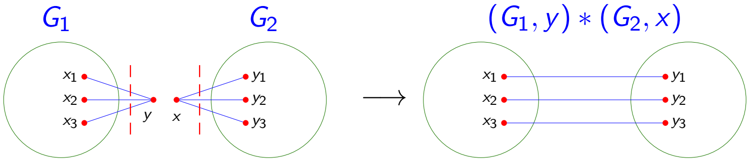

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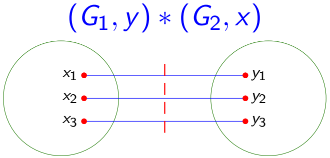

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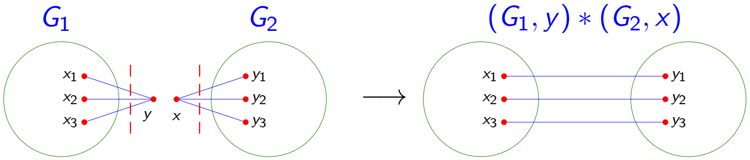

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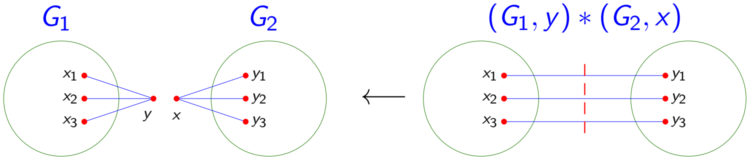

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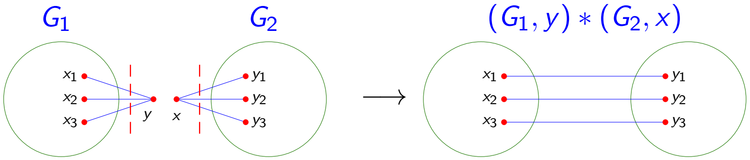

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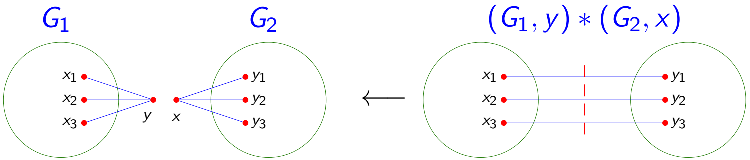

The resulting graphs are called *3–cut reductions* or constituents.

Constructions

Proposition (Funk, Jackson, D.L., Sheehan - JCTB 2003)

If a bipartite graph G can be represented as a star product $G = (G_1, y) * (G_2, x)$, then G is 2–factor hamiltonian if and only if G_1 and G_2 are 2–factor hamiltonian.

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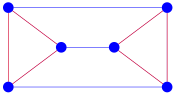

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- (Funk, Jackson, D.L., Sheehan - JCTB 2003): Construction of an infinite family of 2–factor hamiltonian cubic bipartite graphs by taking iterated star products of $K_{3,3}$ and H_0.

Existence results

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There are no k–regular 2–factor hamiltonian bipartite graphs for $k > 3$.

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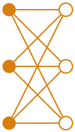
Let G be a 2–factor hamiltonian k-regular bipartite graph. Then either $k = 2$ and G is a circuit or $k = 3$ and G can be obtained from $K_{3,3}$ and H_0 by repeated star products.



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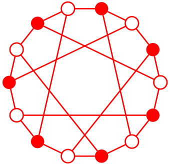

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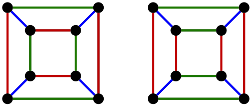

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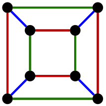

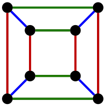

- (Funk, D.L. - Discrete Math. 2000): Let G be a minimally 1–factorable k–regular bipartite graph. Then $k \leq 3$.

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Remark

A smallest counterexample to our Conjecture is cubic and cyclically 4-edge connected *i.e. its 3–cut reductions have no non–trivial 3–edge cuts (D.L. - Discrete Math. 2001), and that it has girth at least six (D.L. - Discrete Math 2002).*

Further results and conjectures

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Faudree, Gould, Jacobson; (2004): Determine the maximum number of edges in 2–factor hamiltonian (bipartite) graphs.

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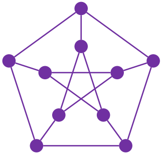

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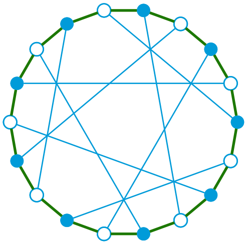

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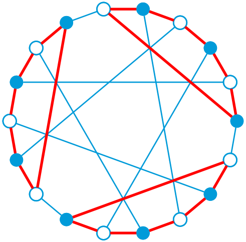

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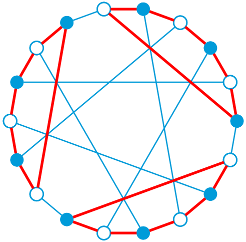

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Theorem (Abreu, Diwan, Jackson, DL, Sheehan - JCTB 2008)

Let G be a pseudo 2-factor-isomorphic cubic bipartite graph. Then G is non-planar.

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Conjecture (Abreu, Diwan, Jackson, DL, Sheehan - JCTB 2008)

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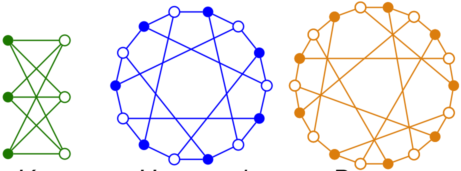

$K_{3,3}$ *Heawood* *Pappus*

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Let G be an essentially 4–edge–connected pseudo 2–factor isomorphic cubic bipartite graph. Then $G \in \{K_{3,3}, H_0, P_0\}$.

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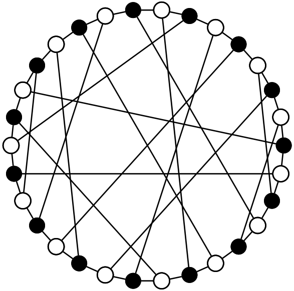

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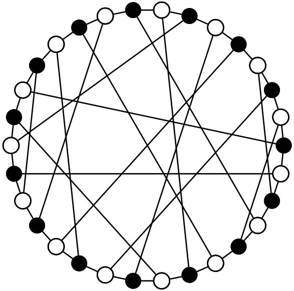

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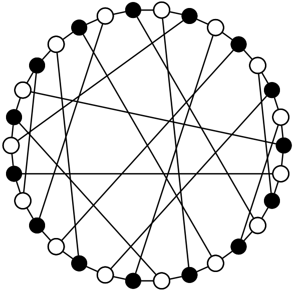

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- *The counterexample $\mathcal{G}$ has order* 30 *and is not* 2*–factor hamiltonian.*
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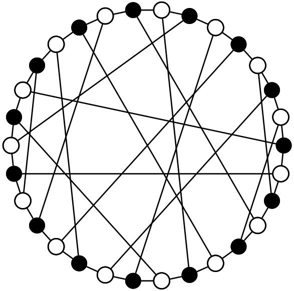

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- $\mathcal{G}$ *has cyclic edge–connectivity* 6*,* $|Aut(\mathcal{G})| = 144$*, is not vertex–transitive.*
- $\mathcal{G}$ *has* 312 *2–factors and the cycle sizes of its 2–factors are* $(6, 6, 18)$*,* $(6, 10, 14)$*,* $(10, 10, 10)$ *and* (30)*.*

Existence: Non–bipartite graphs

Theorem (Abreu, Aldred, Funk, Jackson, DL, Sheehan - JCTB 2004/2009)

Let D be a digraph with n vertices and X be a directed 2–factor of D. Suppose that either

a) $d^{+}(v) \geq \lfloor \log_2 n \rfloor$ *for all* $v \in V(D)$, *or*

b) $d^{+}(v) = d^{-}(v) \geq 4$ *for all* $v \in V(D)$

Then D has a directed 2–factor $Y \ncong X$.

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Suppose that either
(a) $d(v) \geq 2(\lfloor \log_2 n \rfloor + 2)$ *for all* $v \in V(G)$, *or*
(b) G is a $2k$–regular graph for some $k \geq 4$.
Then G has a 2–factor Y with $Y \ncong X$.

Existence: Non–bipartite pseudo 2-factor isomorphic regular graphs

- Let $PU(k)$ (resp. $DPU(k)$) be the class of k–regular pseudo 2–factor isomorphic (resp. directed) graphs.

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Then D has a directed 2–factor Y with different parity of number of circuits from X.

Existence: Non–bipartite pseudo 2-factor isomorphic regular graphs

- Let $PU(k)$ (resp. $DPU(k)$) be the class of k–regular pseudo 2–factor isomorphic (resp. directed) graphs.

Theorem (Abreu, DL, Sheehan - 2010)

Let D be a digraph with n vertices and X be a directed 2–factor of D. Suppose that either

a) $d^+(v) \geq \lfloor log_2\ n \rfloor$ *for all* $v \in V(D)$, *or*

b) $d^+(v) = d^-(v) \geq 4$ *for all* $v \in V(D)$

Then D has a directed 2–factor Y with different parity of number of circuits from X.

Corollary (Abreu, DL, Sheehan - 2009)

- $DPU(k) = \emptyset$ *for* $k \geq 4$;
- *If* $D \in DPU$ *then* D *contains a vertex of out–degree at most* $\lfloor log_2\ n \rfloor - 1$.

Existence: Non–bipartite pseudo 2-factor isomorphic regular graphs

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Theorem (Abreu, DL, Sheehan - 2010)

Let G be a graph with n vertices and X be a 2–factor of G. Suppose that either

a) $d(v) \geq \lfloor log_2 n \rfloor$ *for all* $v \in V(G)$, *or*

b) G *is a* $2k$*–regular graph for some* $k \geq 4$.

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Corollary (Abreu, DL, Sheehan - 2009)

- *If $G \in PU$ then G contains a vertex of degree at most* $2\lfloor log_2\ n \rfloor + 3$.
- $PU(2k) = \emptyset$ *for* $k \geq 4$.

Open problems

Question

Do there exist 2–factor isomorphic bipartite graphs of arbitrarily large minimum degree?

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Conjecture (Abreu, Aldred, Funk, Jackson, D.L., Sheehan; JCTB 2004)

The graph K_5 is the only 2–factor hamiltonian 4–regular non–bipartite graph.

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Is $PU(2k+1) = \emptyset$ *for* $k \geq 2$*? In particular, are* $PU(7)$ *and* $PU(5)$ *empty?*

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Is K_5 *the only* 4*–edge connected graph in* $PU(4)$*?*

Characterization: non–bipartite connected k–regular 2–factor isomorphic graphs, $k \geq 3$

- This class is very small for $k \geq 4$:

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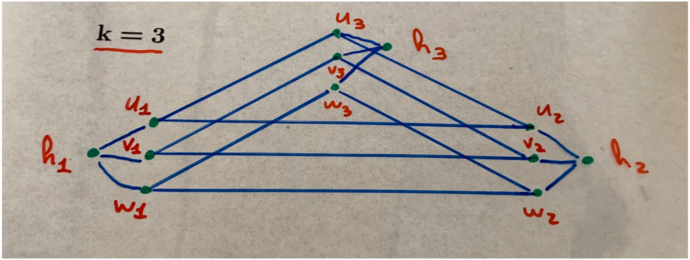
- For $k = 3$ the class of non bipartite k–regular 2–factor hamiltonian graphs is quite rich of examples:

Constructions in the class of non bipartite cubic 2–factor hamiltonian graphs

$A(k)$, $k \geq 3$ is the graph with

- $V = \{h_i, u_i, v_i, w_i : i = 1, 2, \ldots, k\}$
- $E = \{h_i u_i, h_i v_i, h_i w_i, u_i u_{i+1}, v_i v_{i+1}, w_i w_{i+1} : i = 1, 2, \ldots, k\}$ (where the subscript addition is modulo k).



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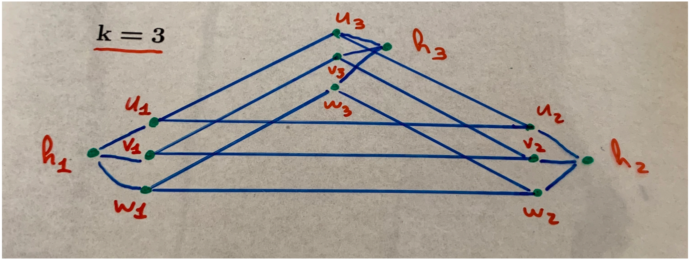

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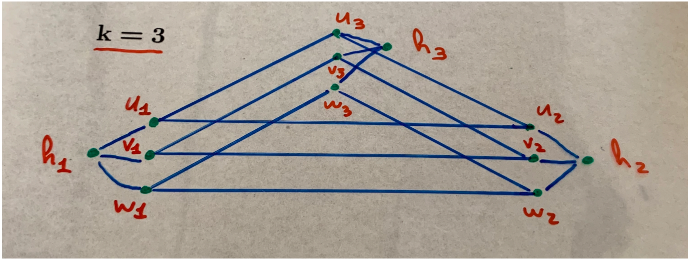

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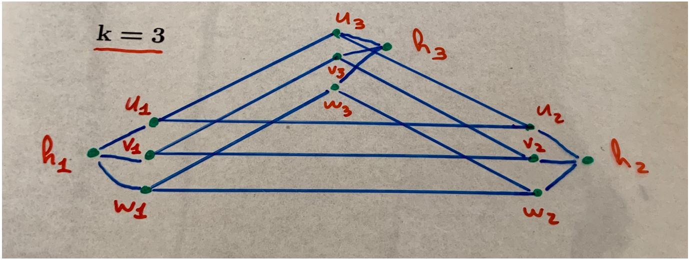

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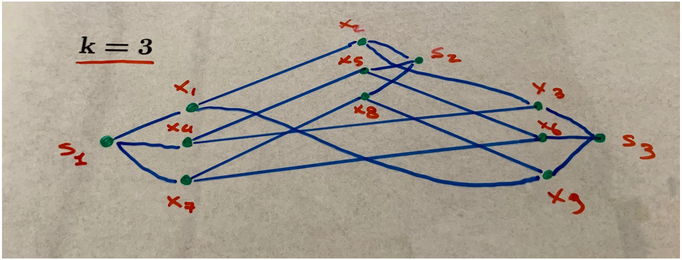
- $A(k)$, $3 \leq k \leq 5$ is cyclically k–edge connected.

Constructions in the class of non bipartite cubic 2–factor hamiltonian graphs

$B(k)$, $k \geq 3$ is the graph with

- $V = \{s_i : i = 1, \ldots, k\} \cup \{x_j \ : \ j = 1, \ldots, 3k\}$
- $E = \{s_i x_i, s_i x_{i+k}, s_i x_{i+2k} : i = 1, \ldots, k\} \cup \{x_j x_{j+1} : j = 1, \ldots, 3k\}$ (where the subscript addition is modulo $3k$).



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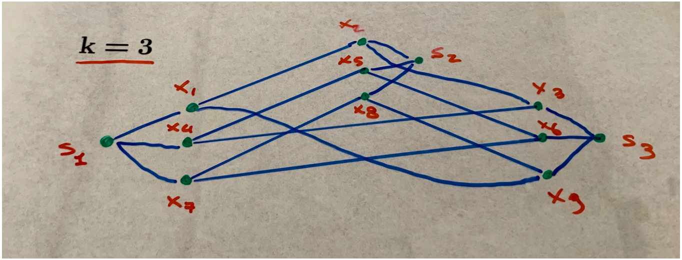

- $B(k)$ is the *twist* of $A(k)$!

Construction in the class of non bipartite cubic 2–factor hamiltonian graphs

Theorem

$A(k), B(K)$, for k odd and $k \geq 3$, provide infinite families of 3–connected cubic 2-factor hamiltonian non–bipartite graphs. These graphs are also maximal.

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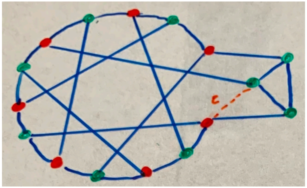

$H_0 * K_4 \in HU(3)$
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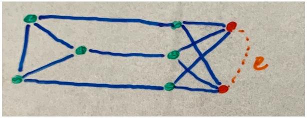

$K_4 * K_{3,3} \in HU(3)$
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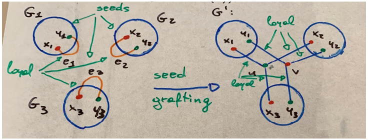
Construction in the class of (non bipartite) cubic 2–factor isomorphic graphs

- *Seed grafting*: G_i cubic bipartite 2–factor hamiltonian. G' cubic bipartite 2–factor isomorphic.



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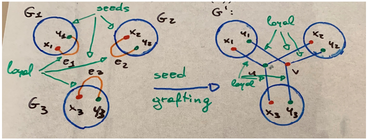

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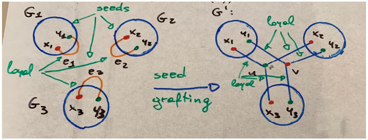
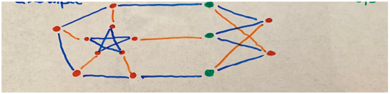


- infinite family of connectivity 2 cubic bipartite 2-factor isomorphic graphs!

Construction in the class of non bipartite cubic 2–factor isomorphic graphs

- The seed grafting with P as a seed is a 'maximal' infinite family of connectivity 2 cubic non–bipartite 2–factor isomorphic.



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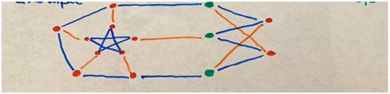
- Star products $P * G$, G cubic bipartite 2–factor hamiltonian are 'maximal' infinite family of 3–connected cubic non–bipartite 2–factor isomorphic graphs.



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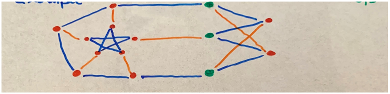
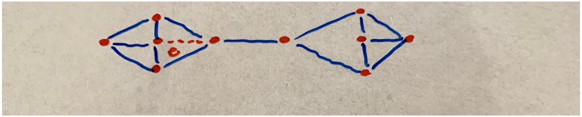


- 2–factor $= C_5 \cup C_9$.

Construction in the class of non bipartite cubic 2–factor isomorphic graphs

- Not all cubic non–bipartite 2–factor isomorphic graphs are 'maximal'.



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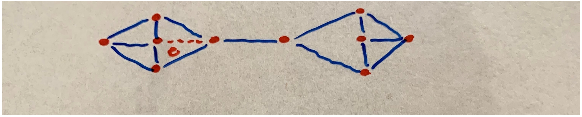

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Conjecture (Aldred, Funk, DL, Jackson, Sheehan; JCTB 2004)

There exists an integer k such that there is no cyclically k–edge connected cubic non bipartite 2–factor isomorphic graph.

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Question

Is there any chance of (partially) characterize these classes of non-bipartite k–regular 2–factor isomorphic/hamiltonian graphs?

THANK YOU