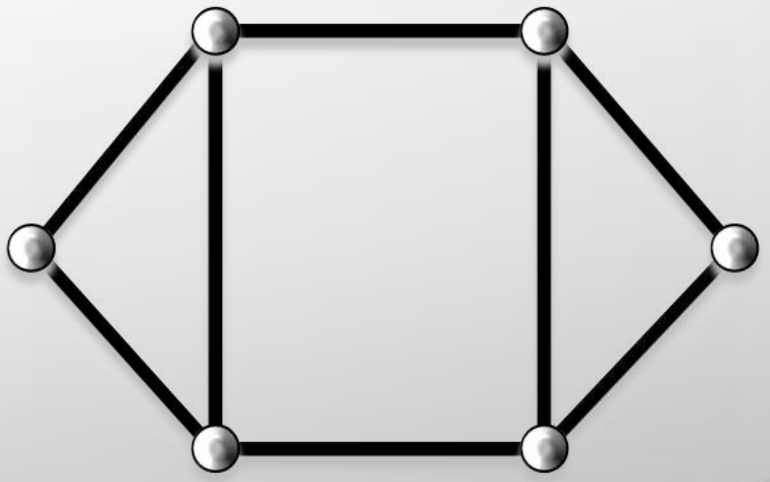
THE 1-2-3 CONJECTURE ALMOST ALMOST HOLDS FOR REGULAR GRAPHS

JAKUB PRZYBYŁO

AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY,

KRAKOW, POLAND

Every non-trivial graph contains a pair of vertices with equal degrees.



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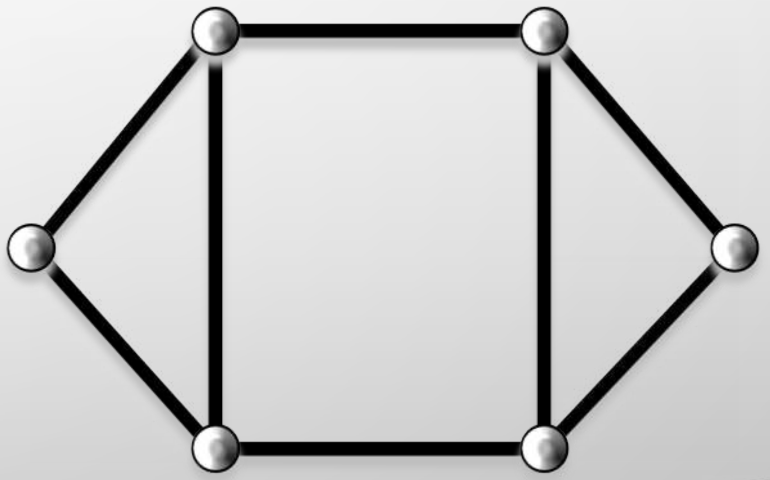

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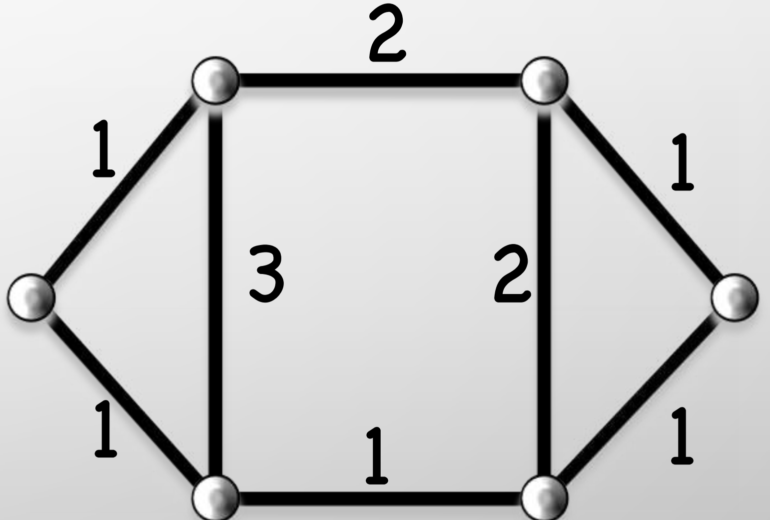

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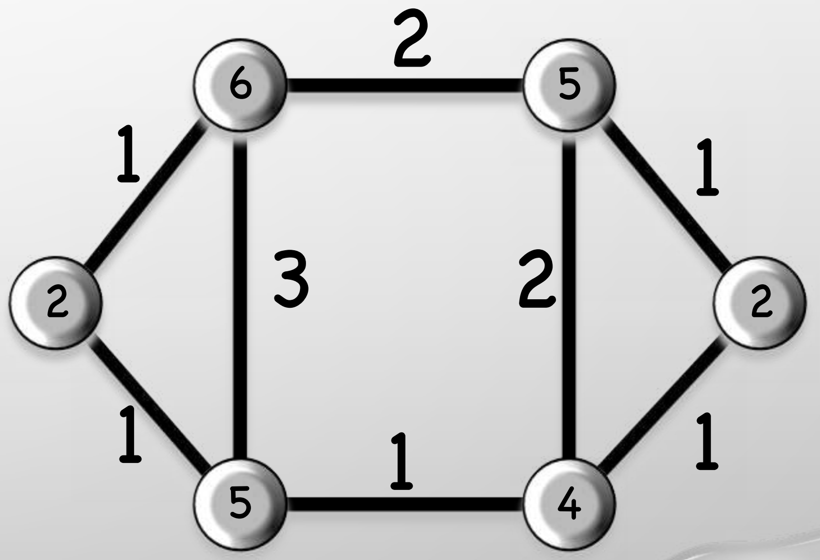

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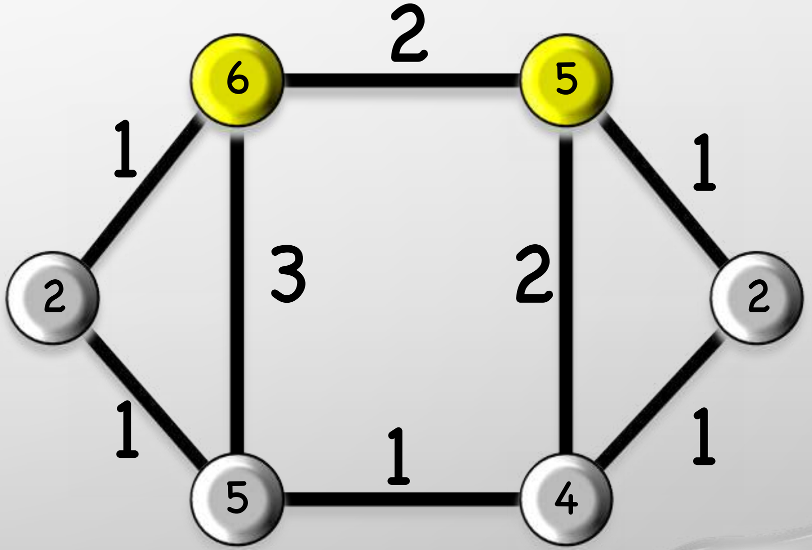

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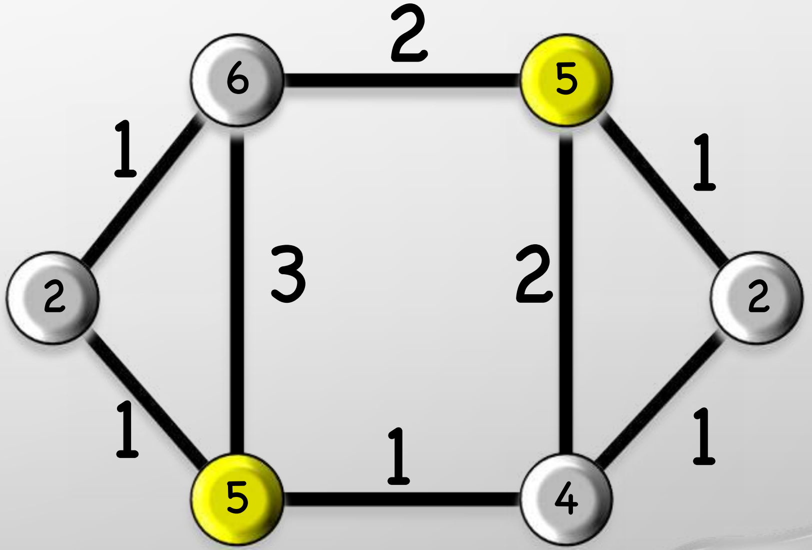

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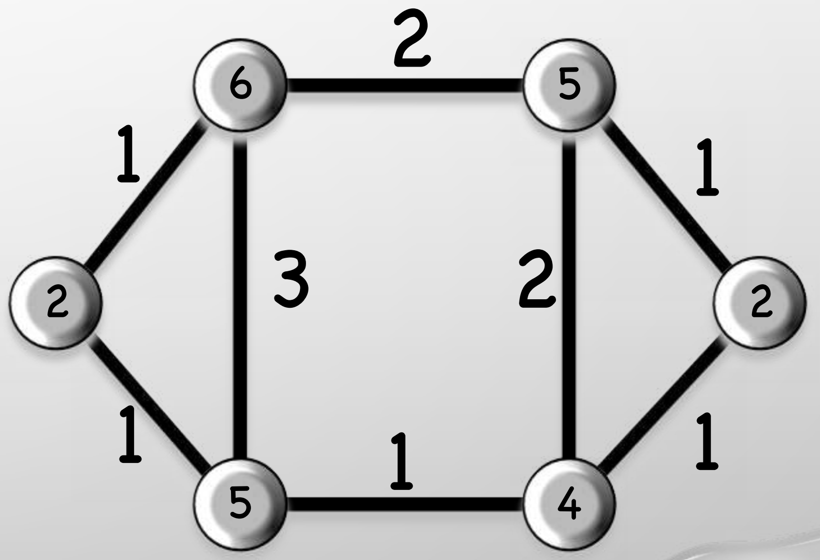

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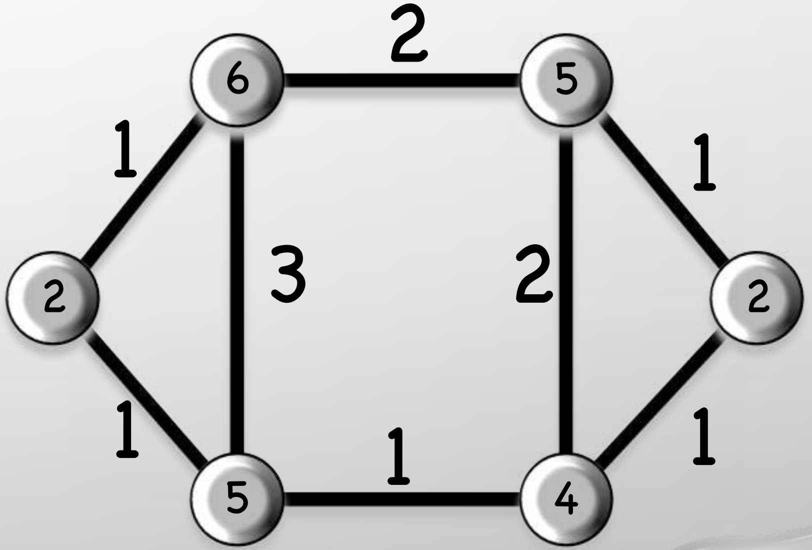

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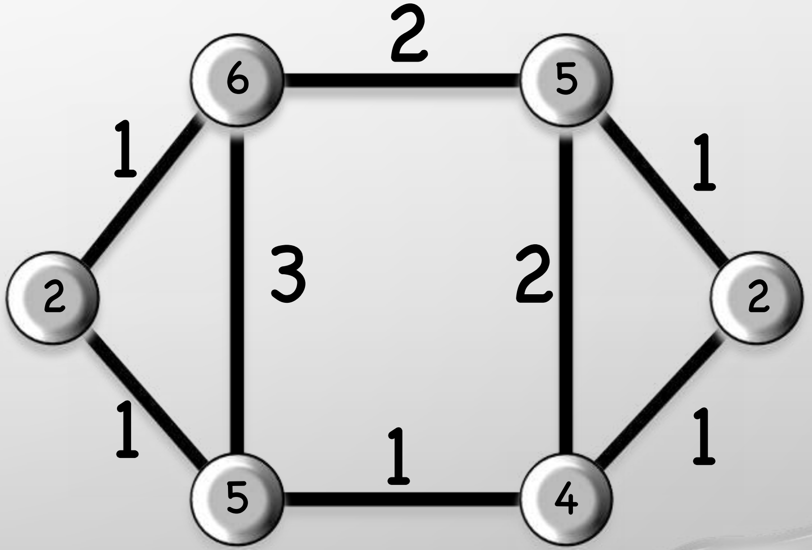

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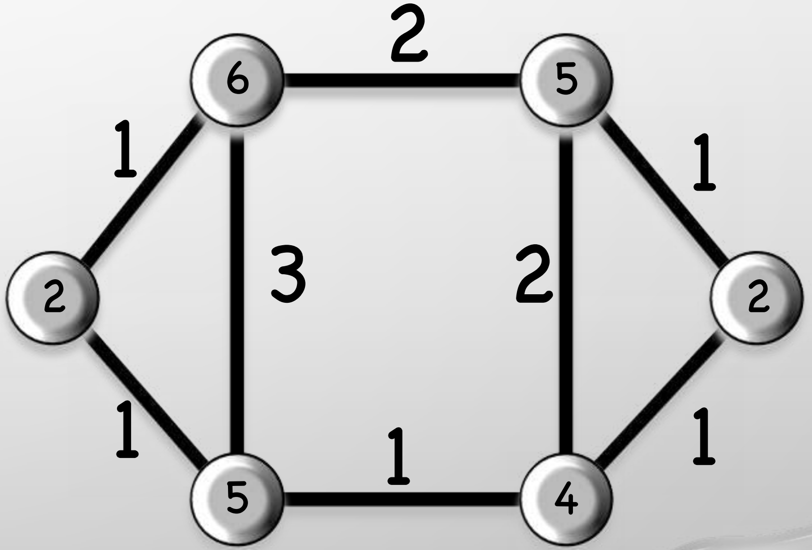

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Th. 1-2-3 Conjecture holds if $\delta(G) > 0.99985\, n$ and n is large enough. (Zhong 2019)

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Every graph is (2,3)-choosable, Combnatorica 36 (Wong, Zhu 2016)

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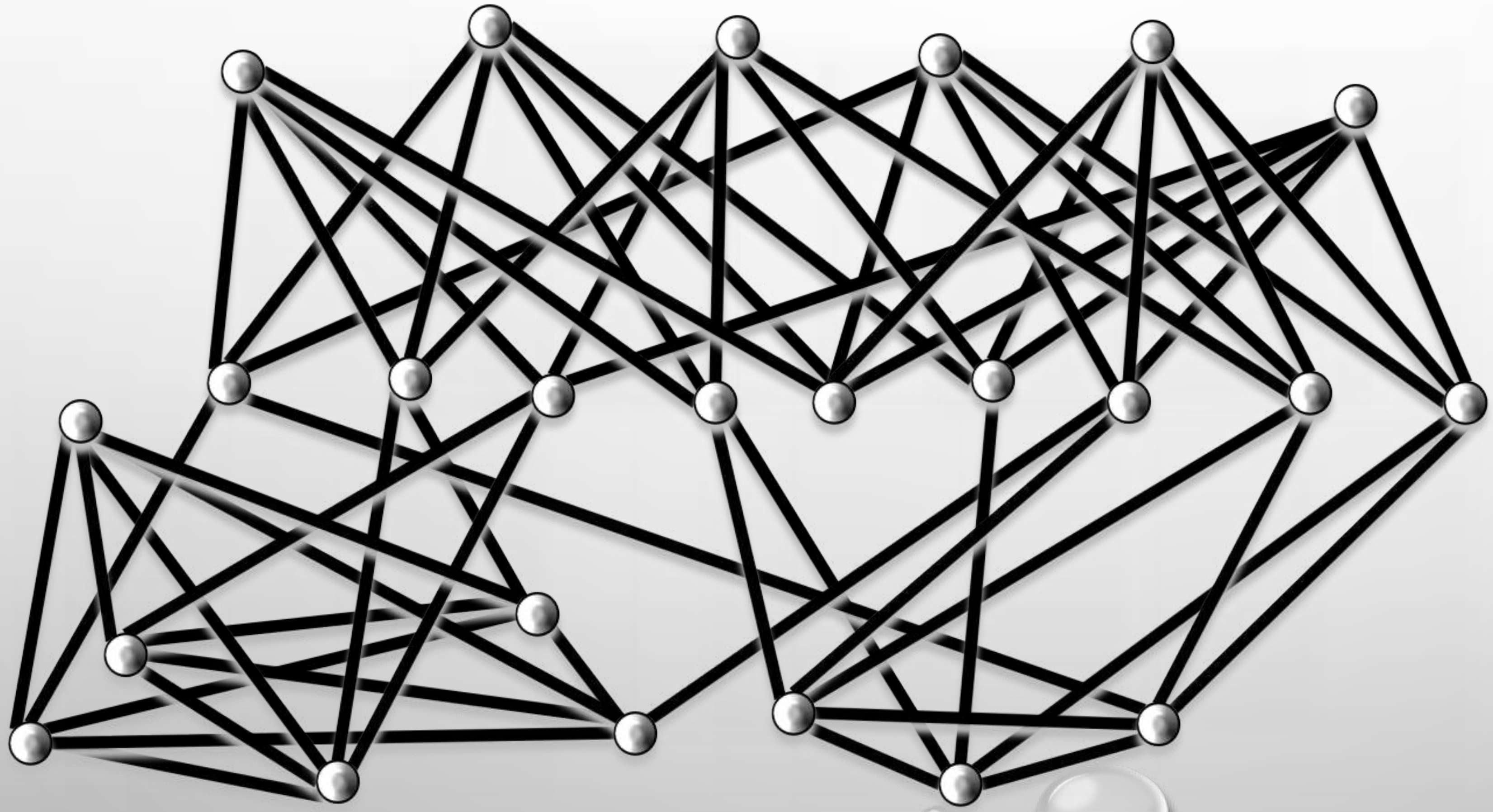
1-2-3-4-colouring of d-regular graphs



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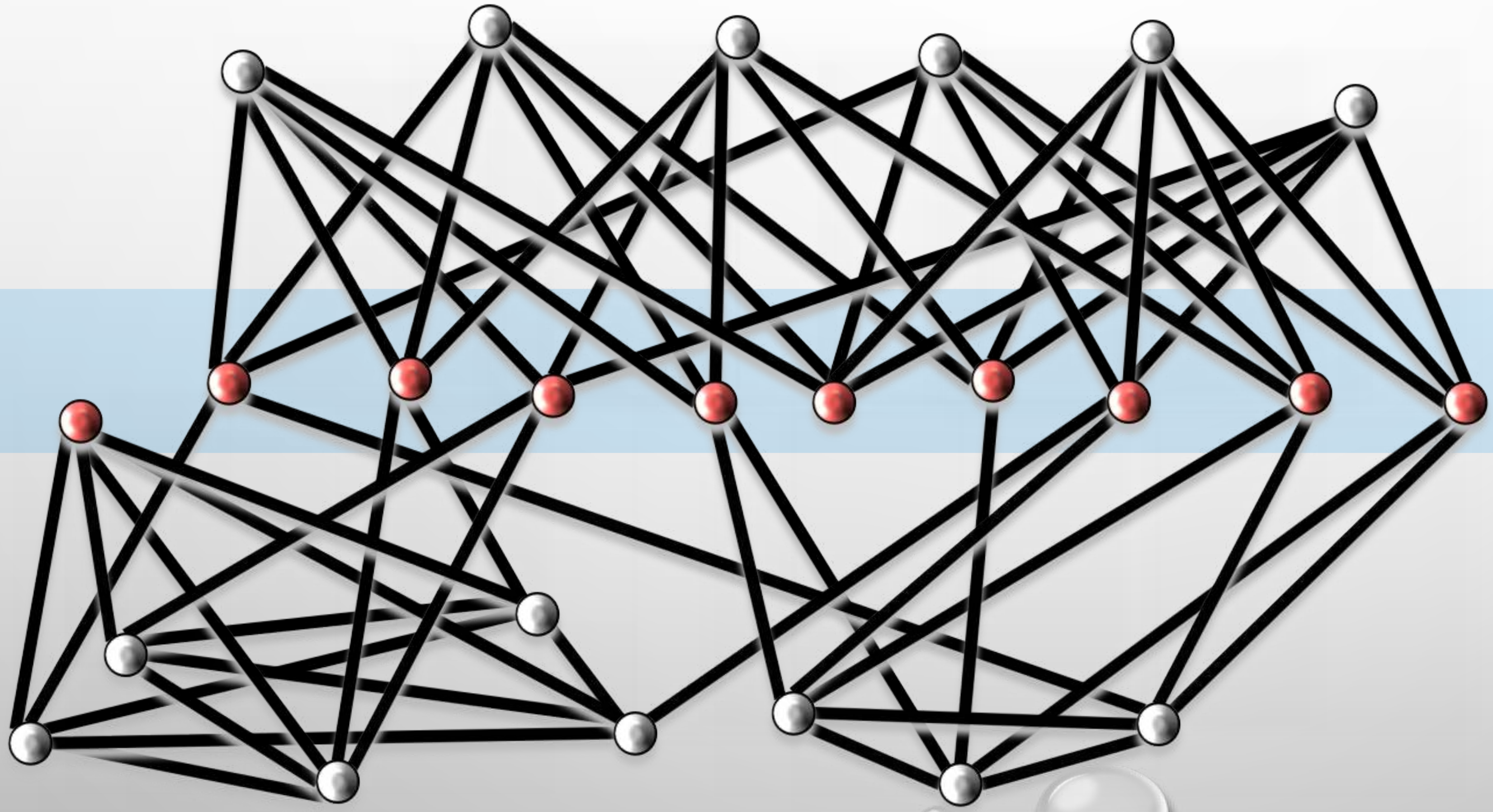

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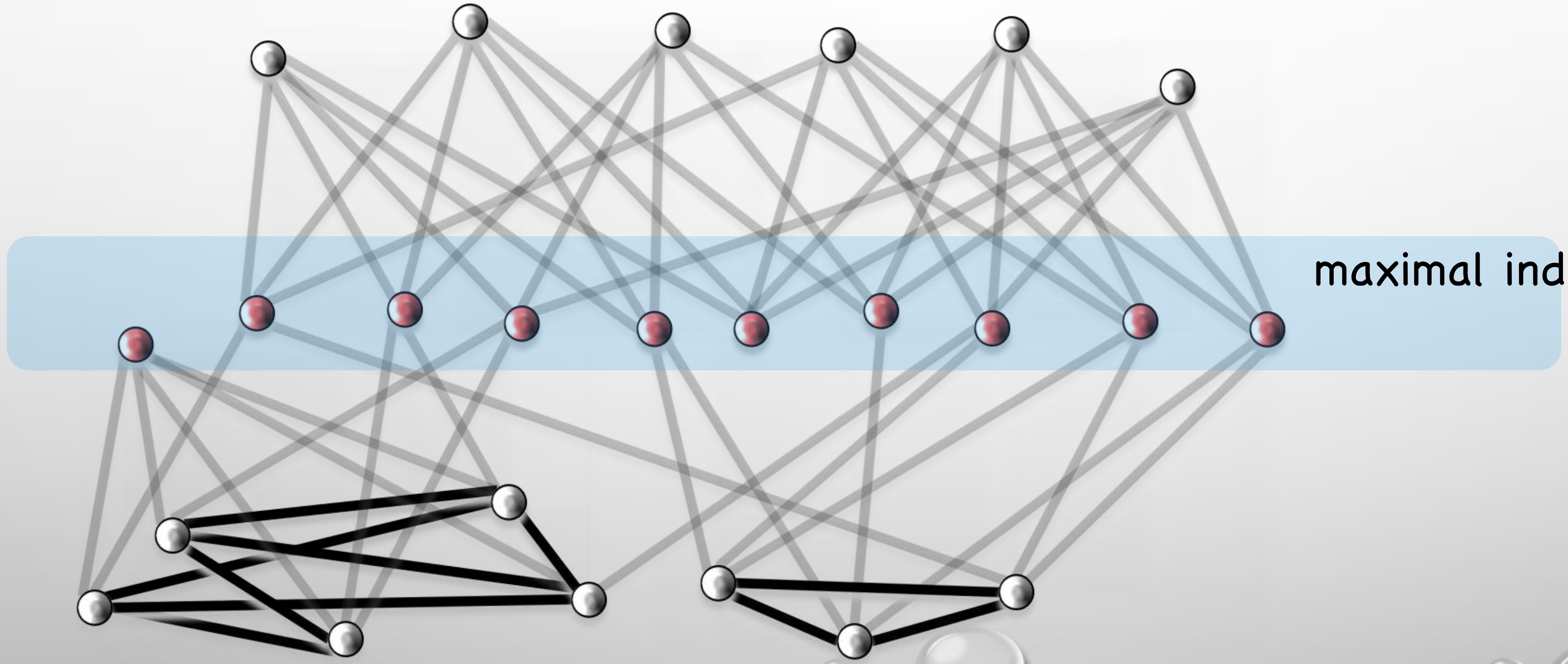

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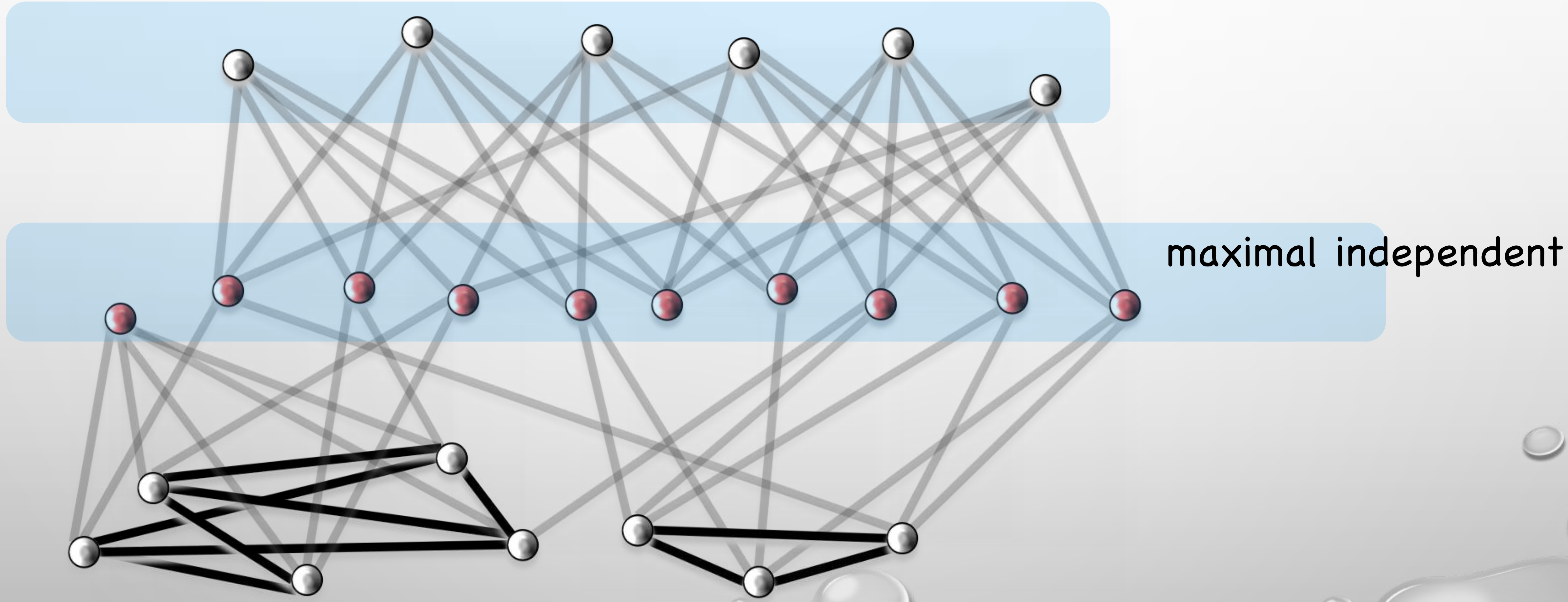

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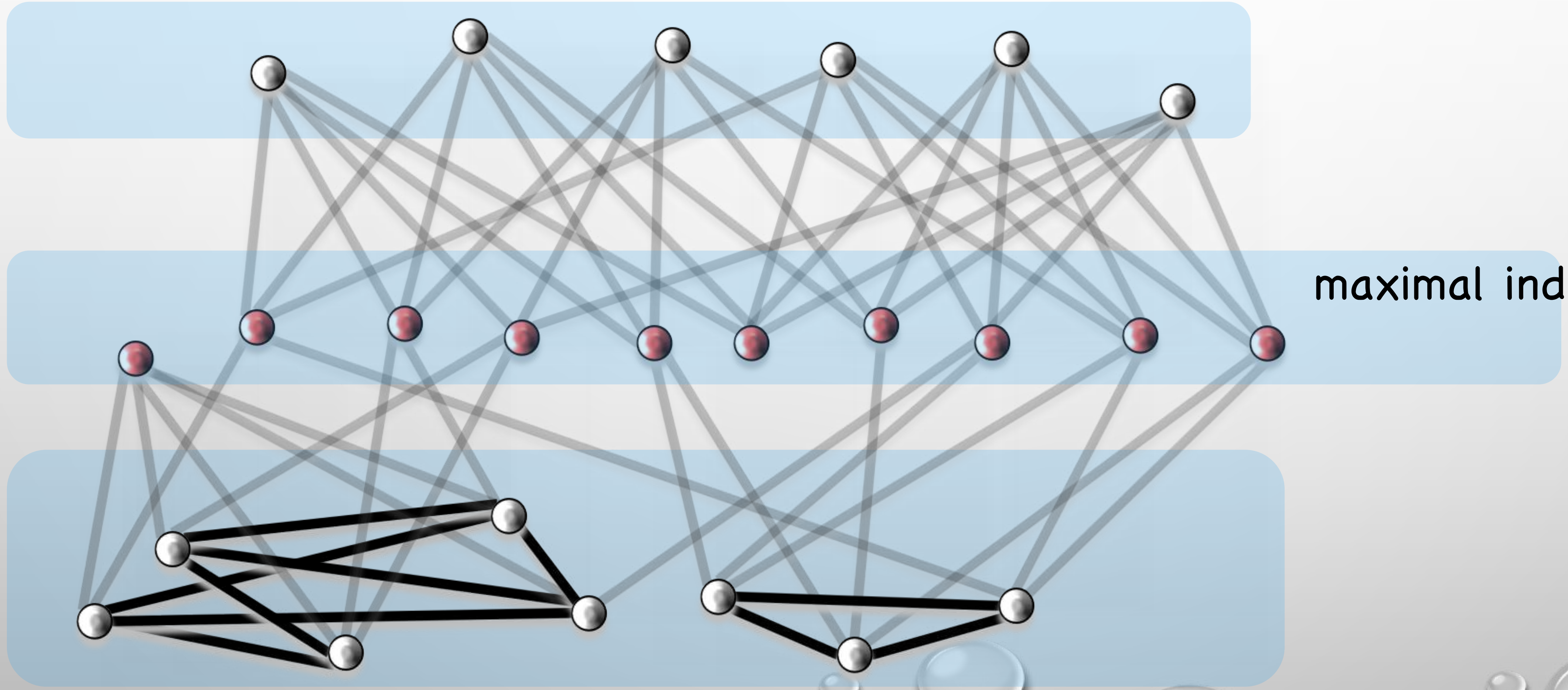

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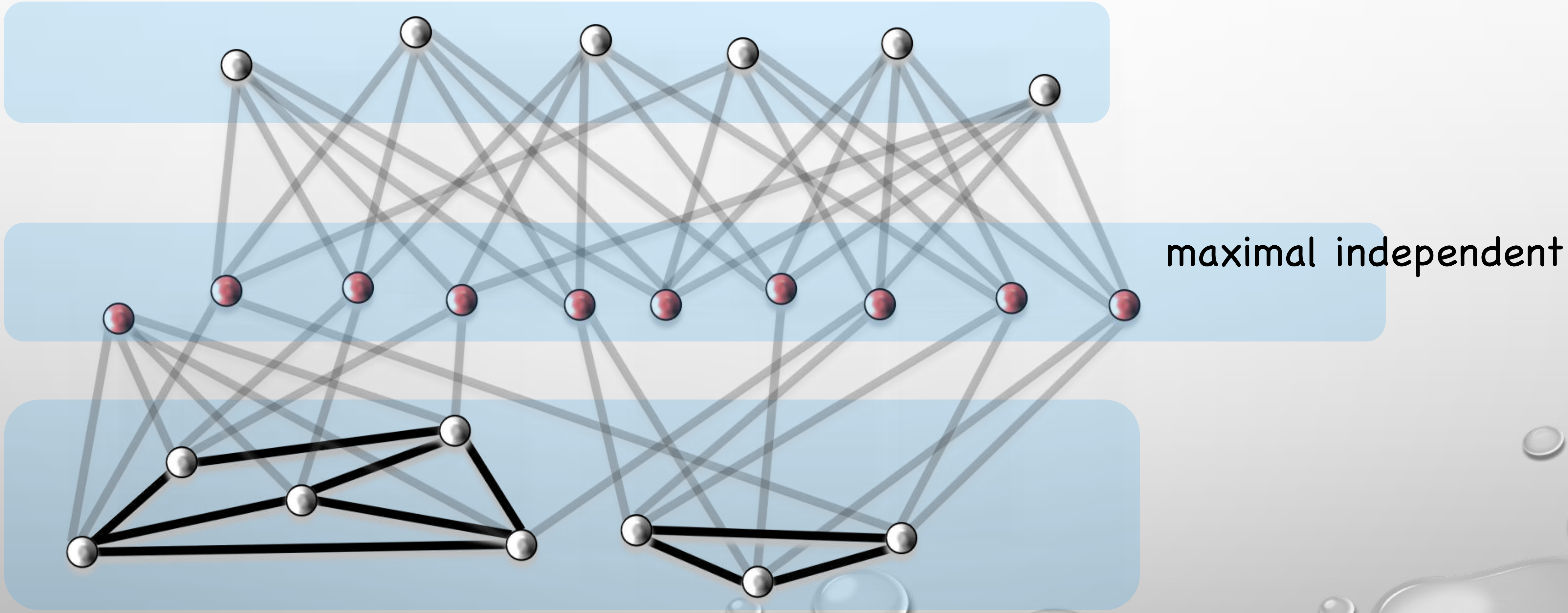

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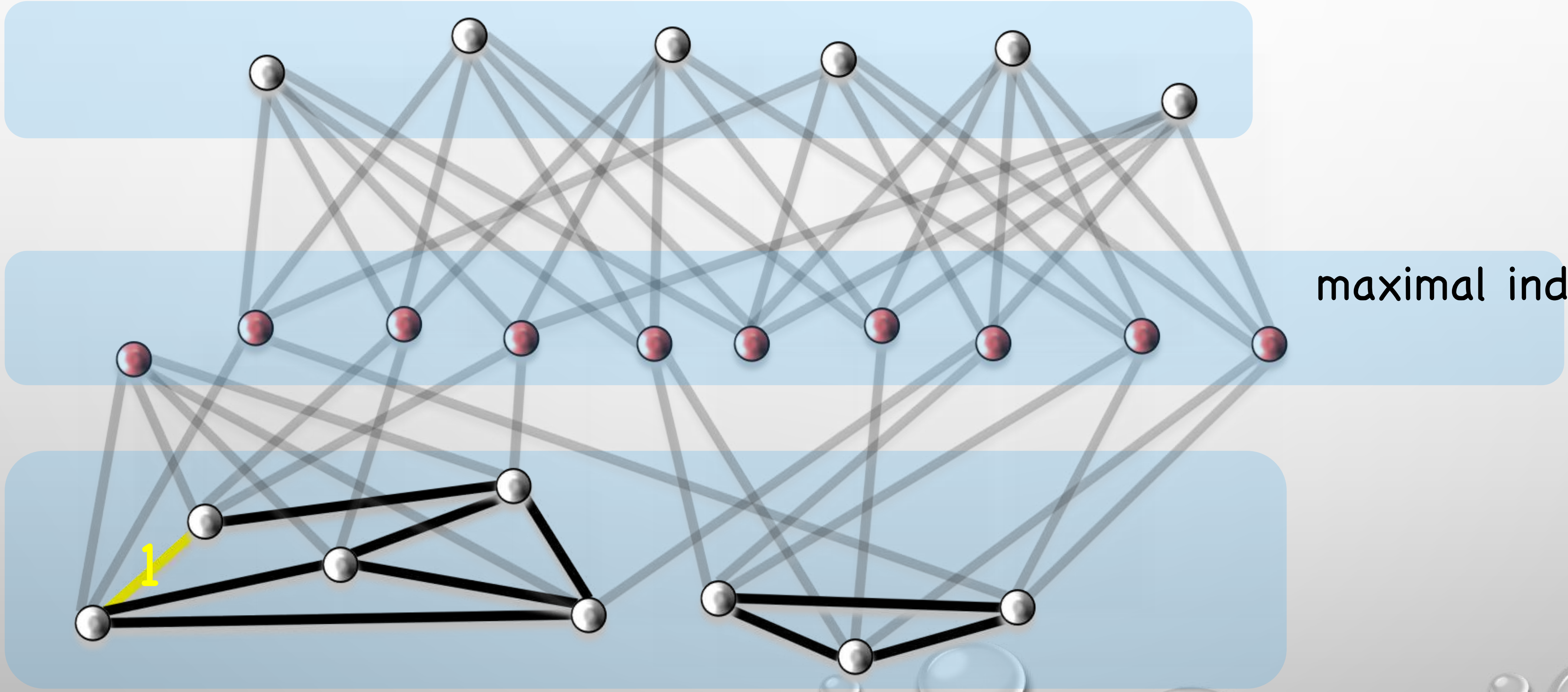

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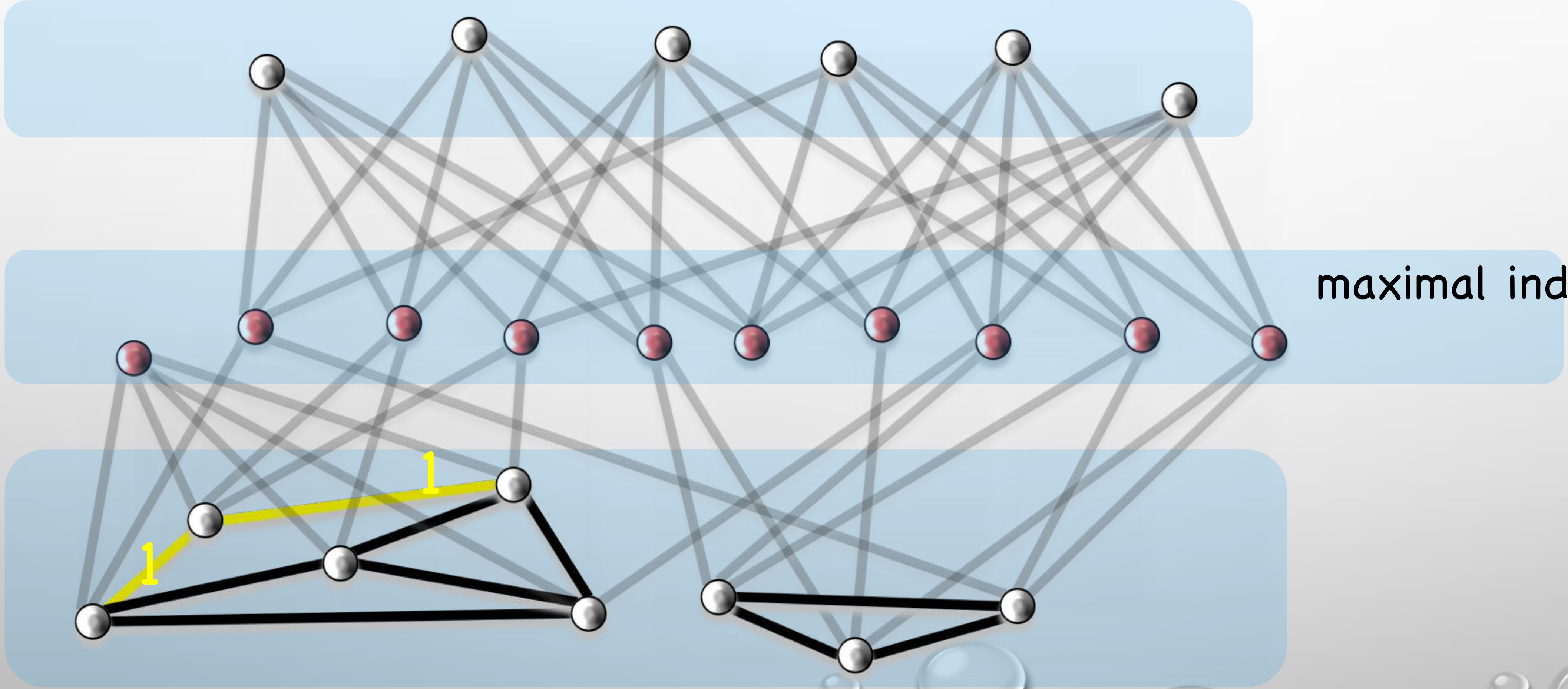

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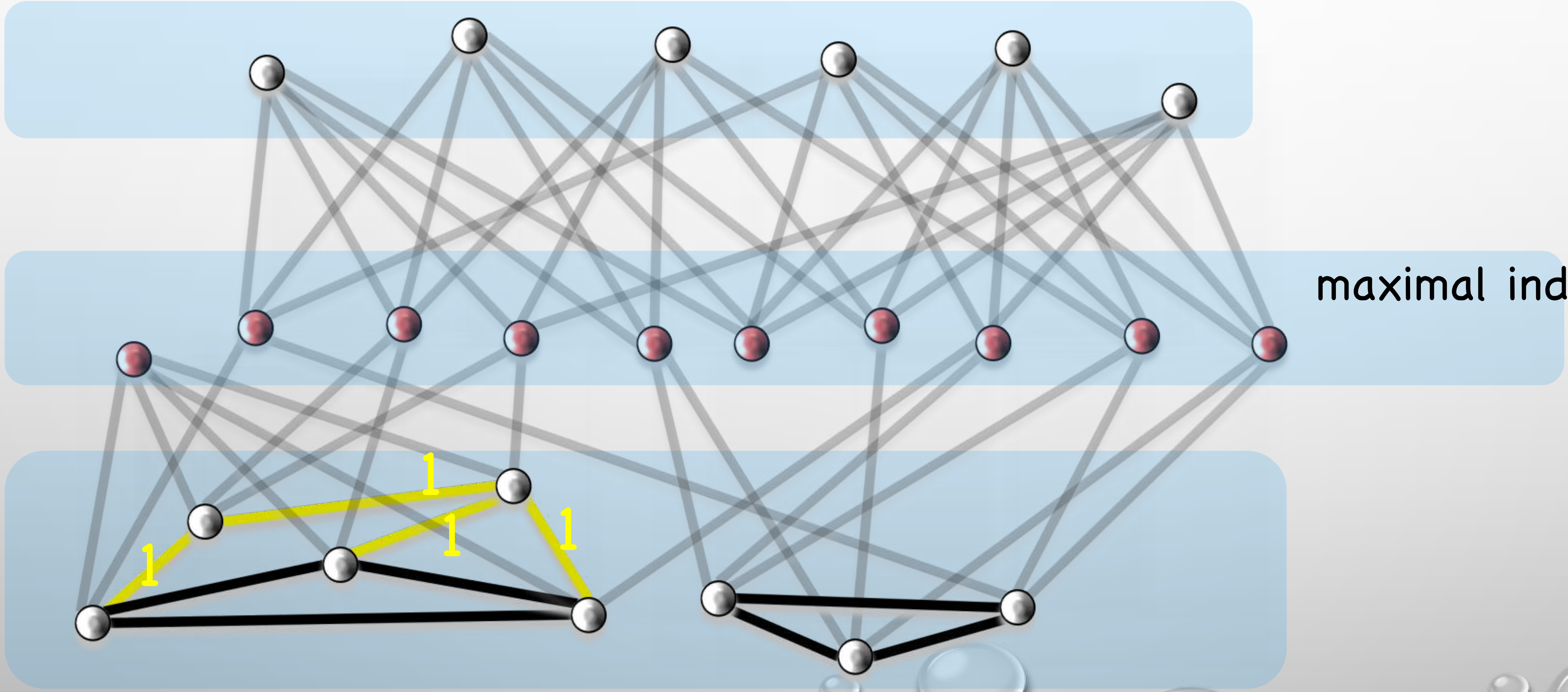

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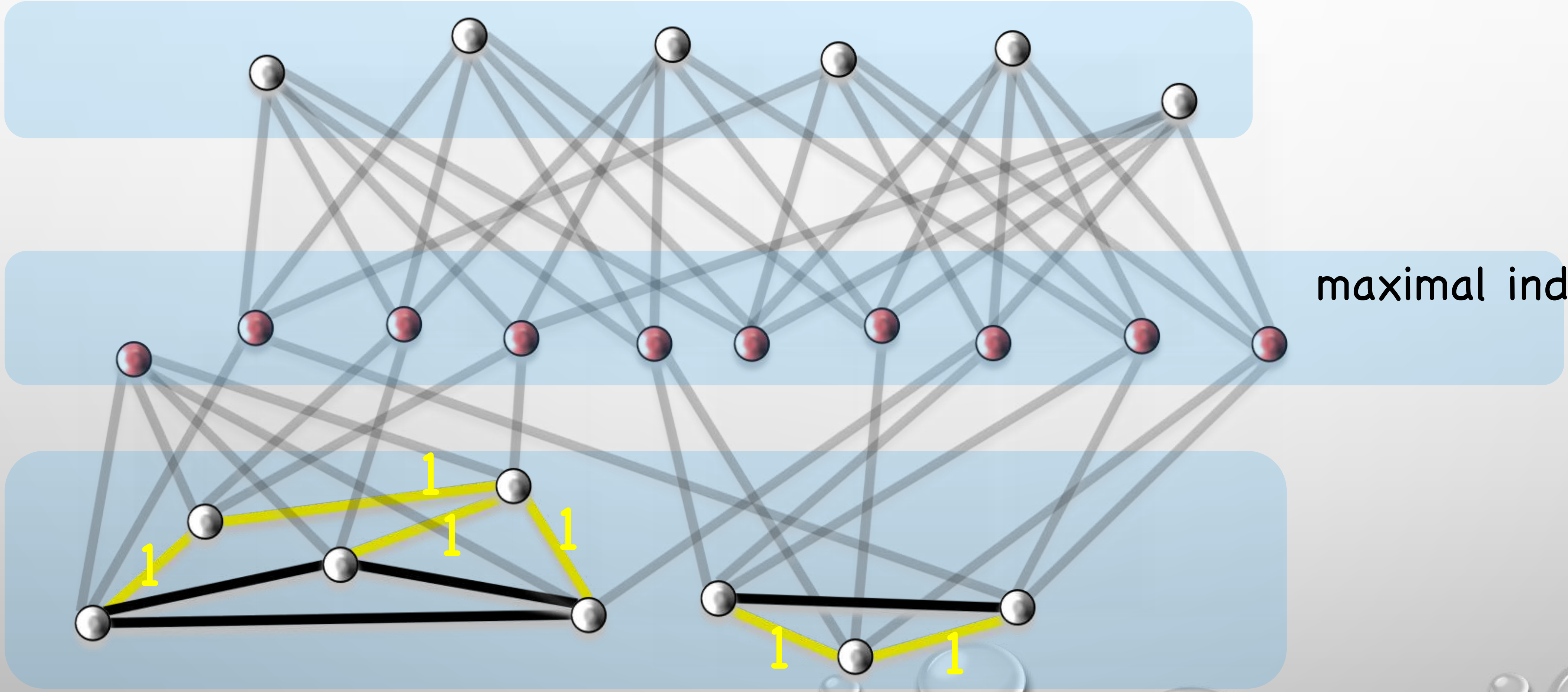

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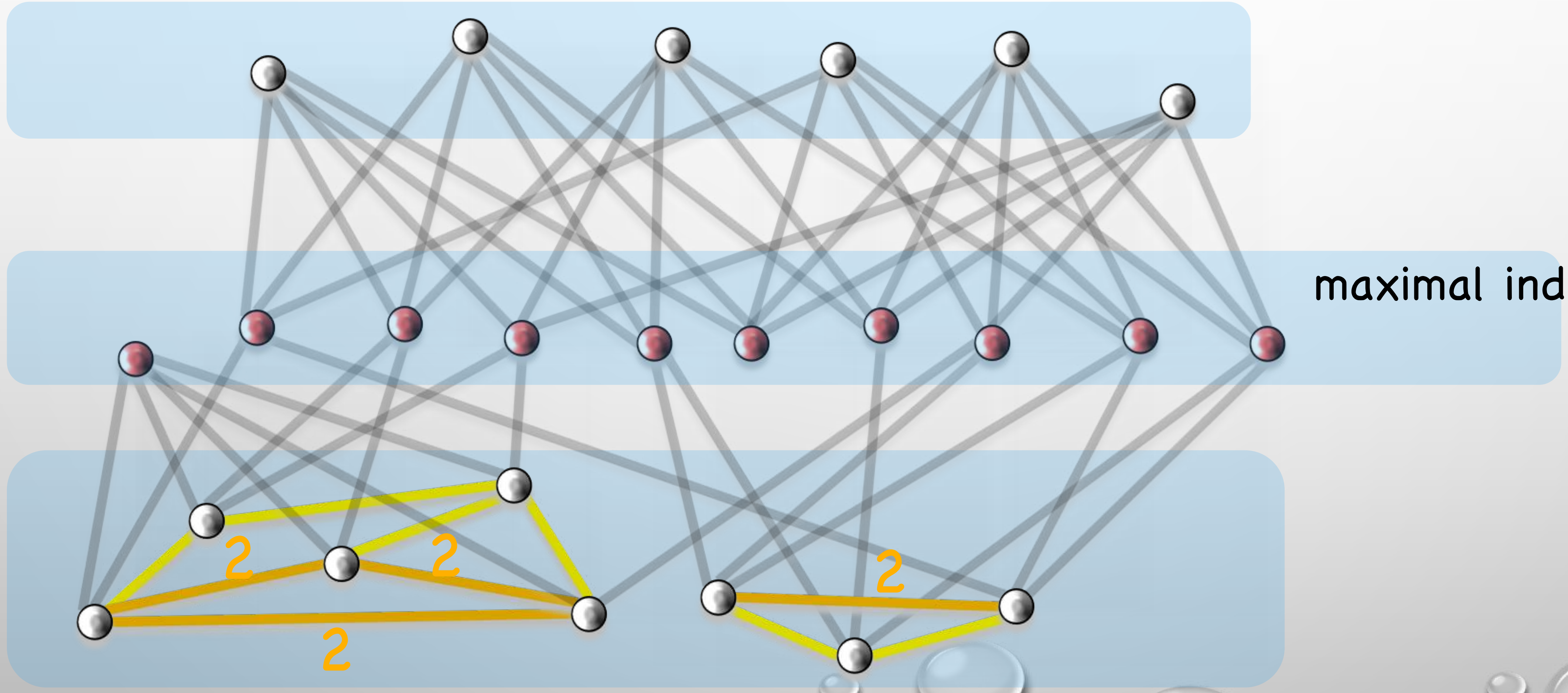

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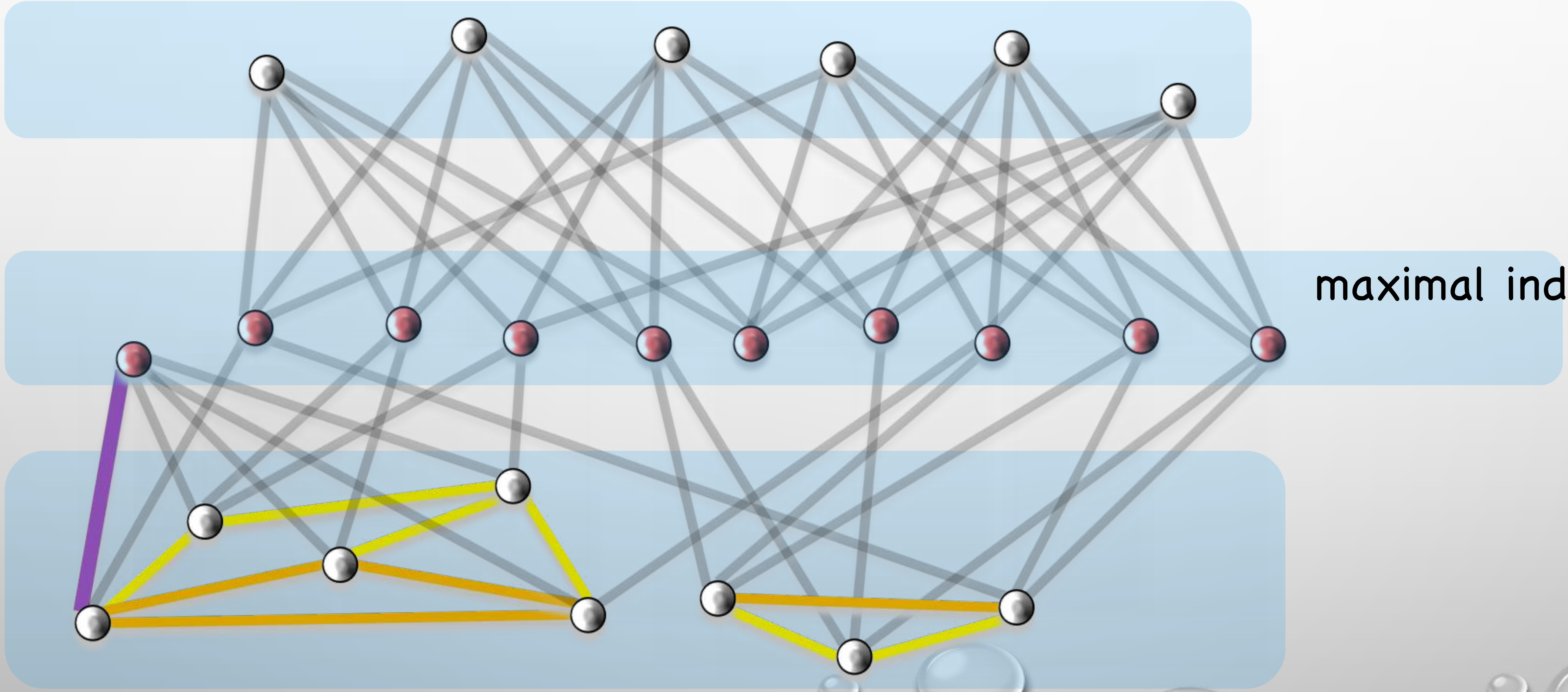

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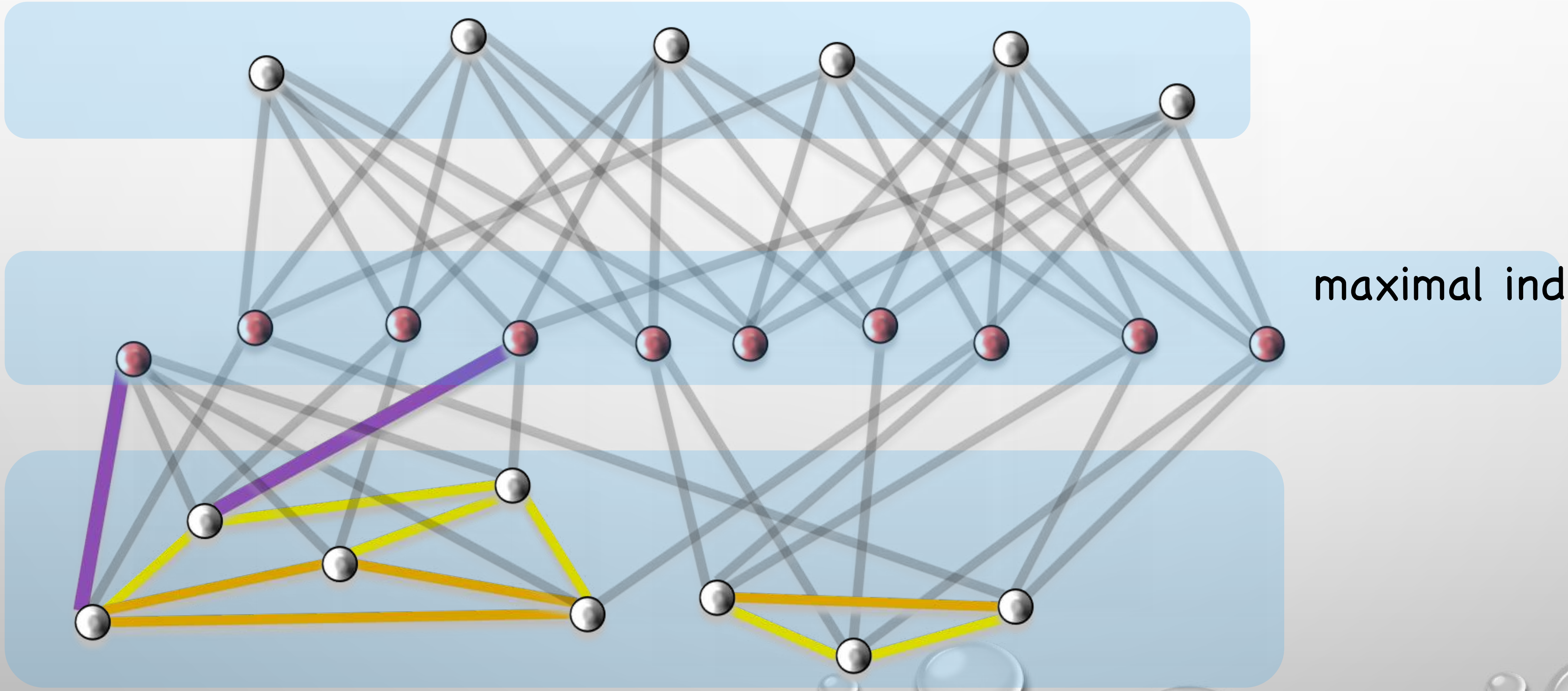

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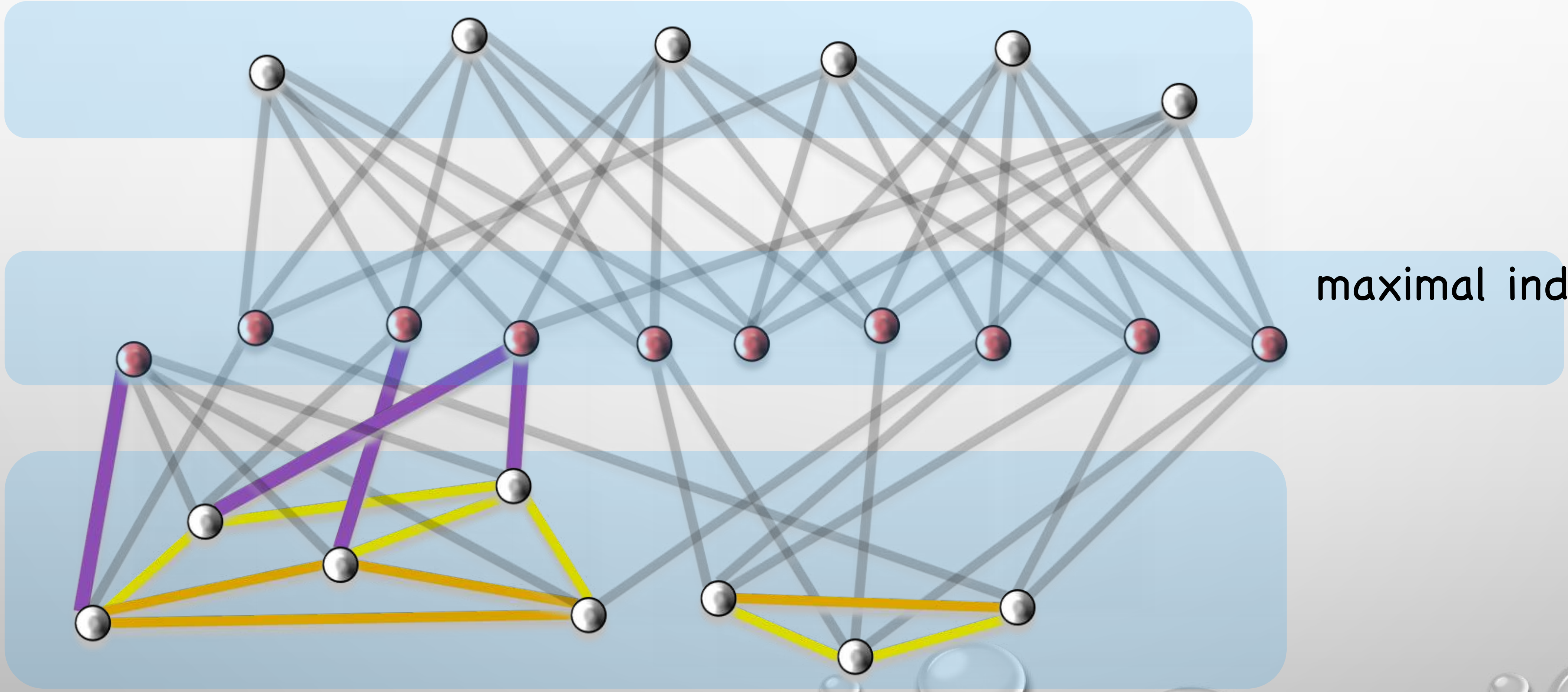

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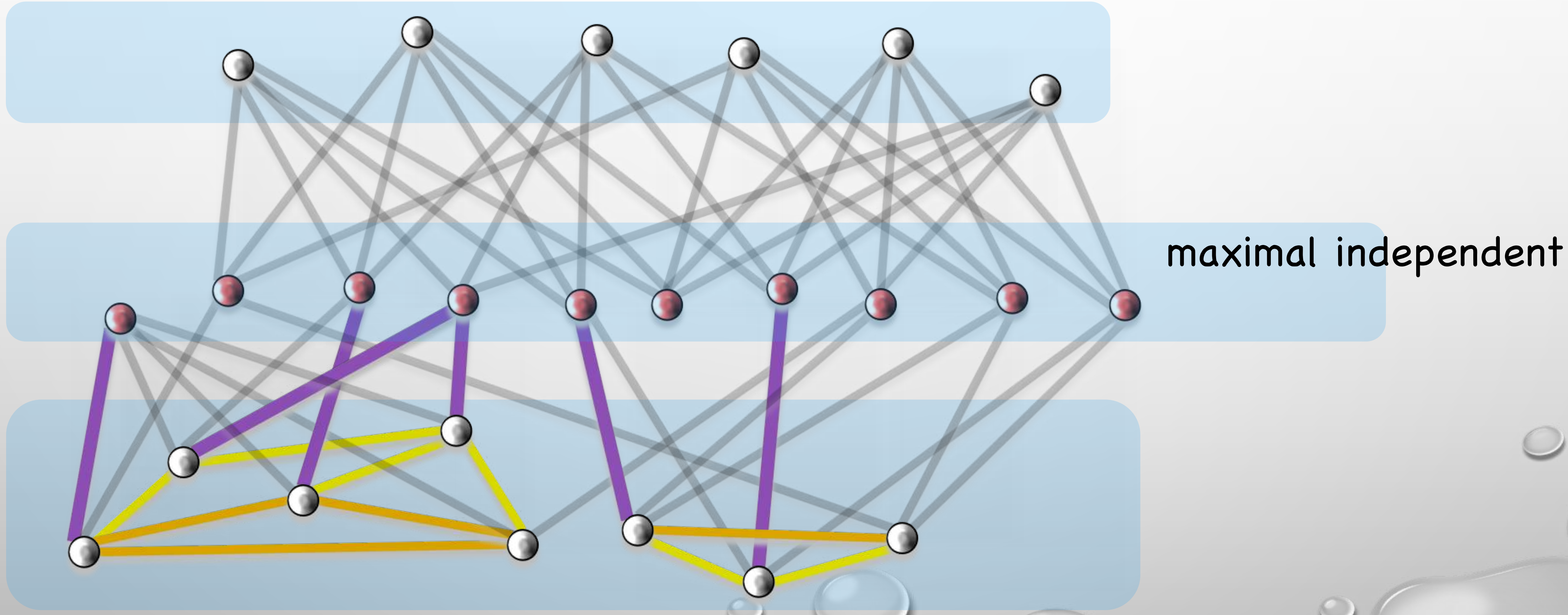

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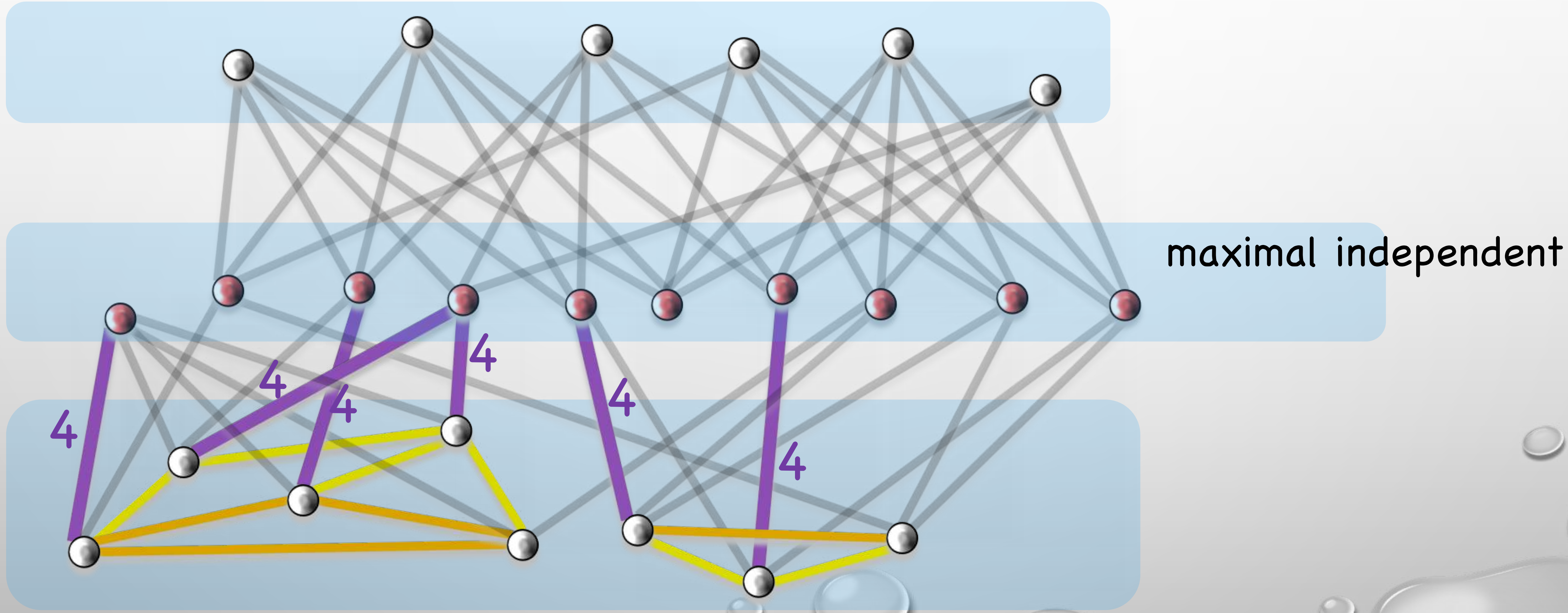

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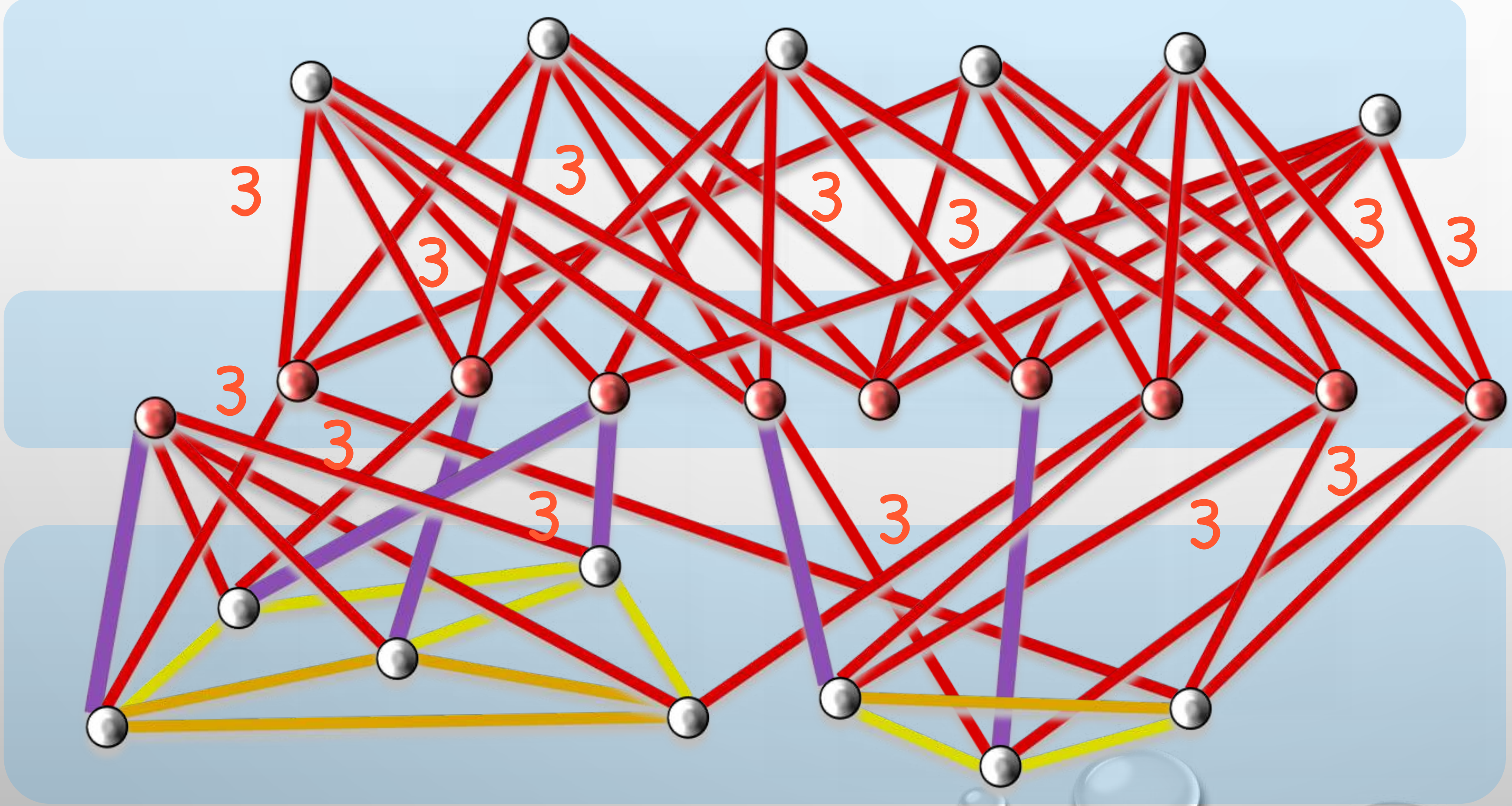

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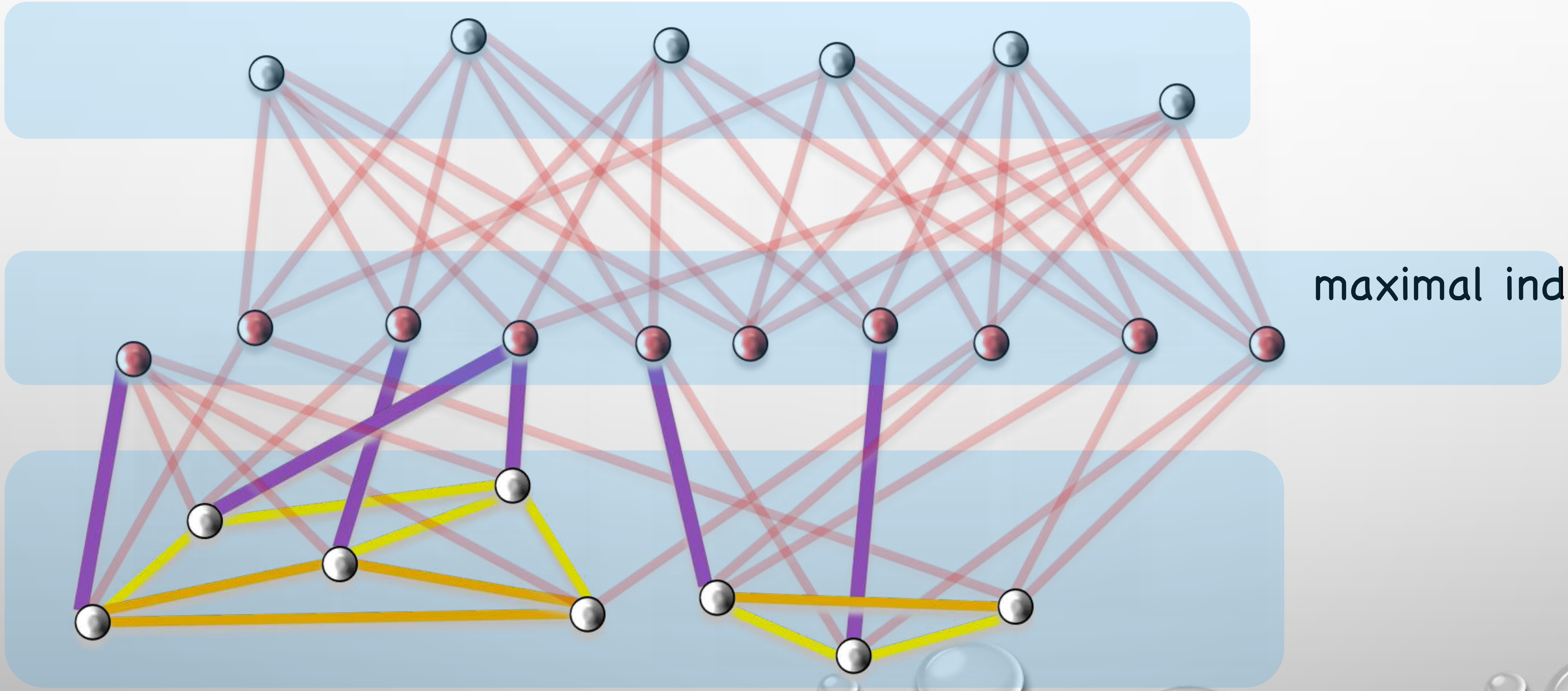

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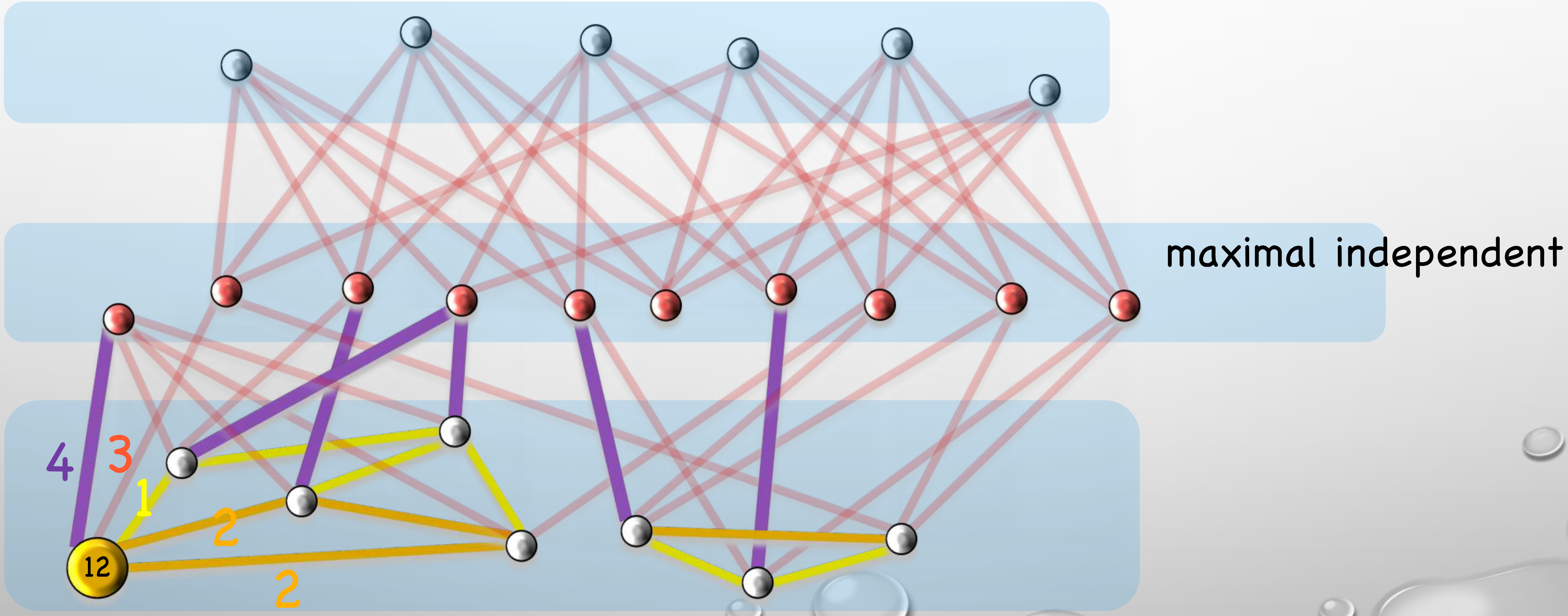

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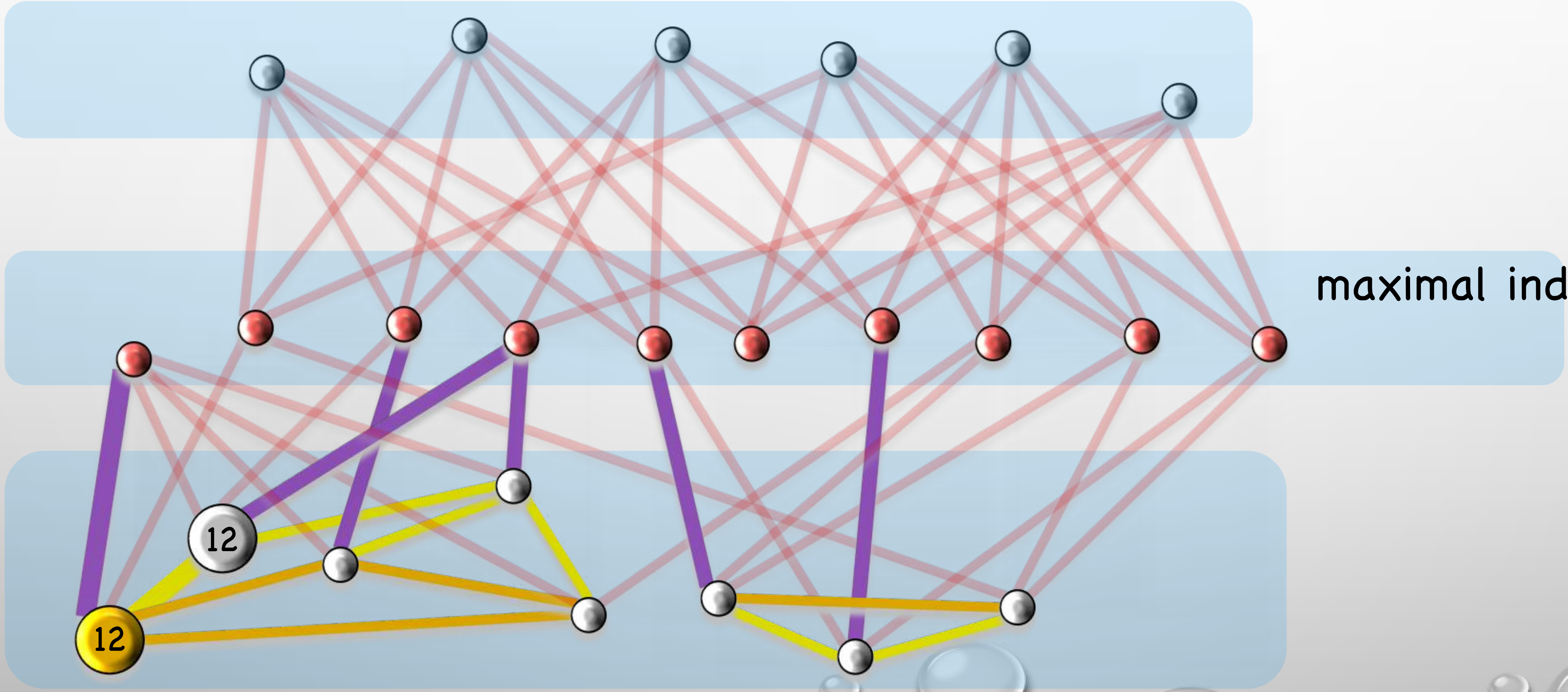

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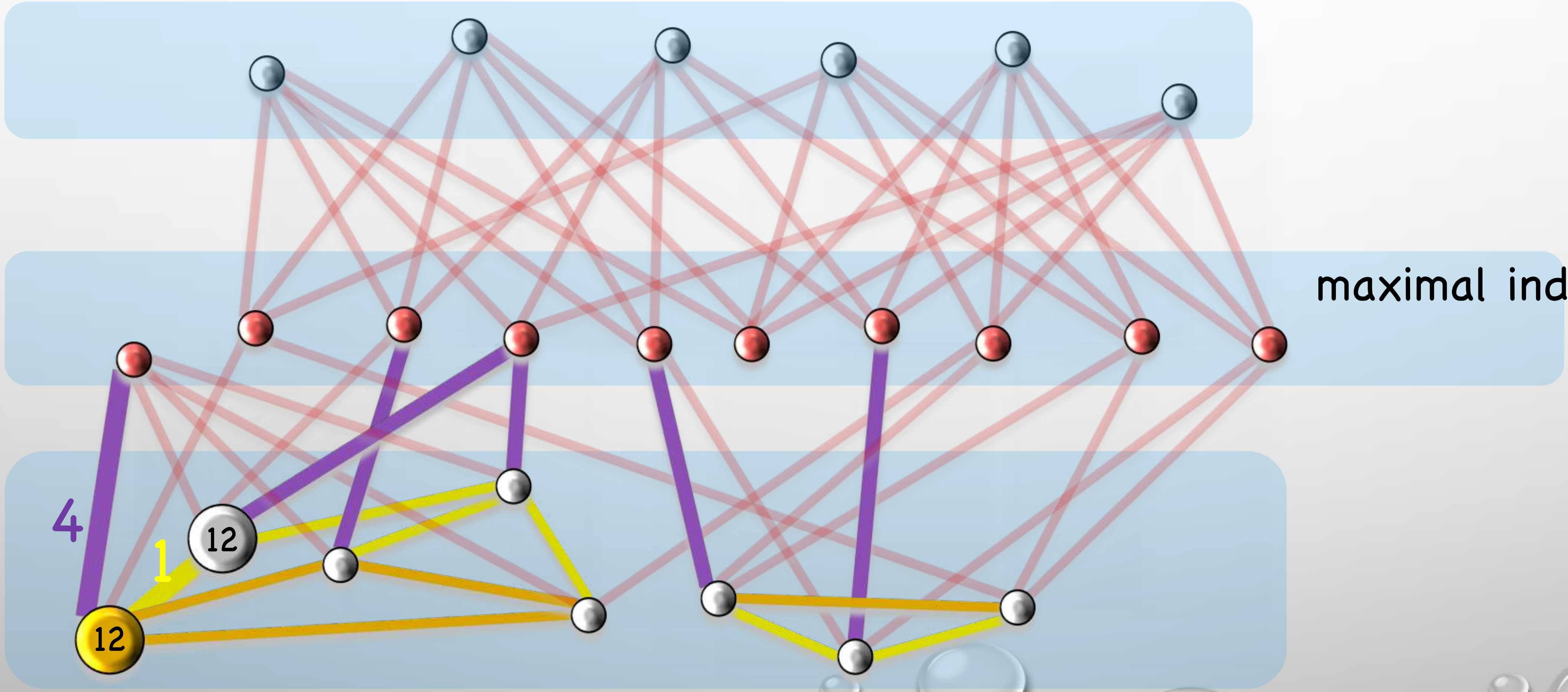

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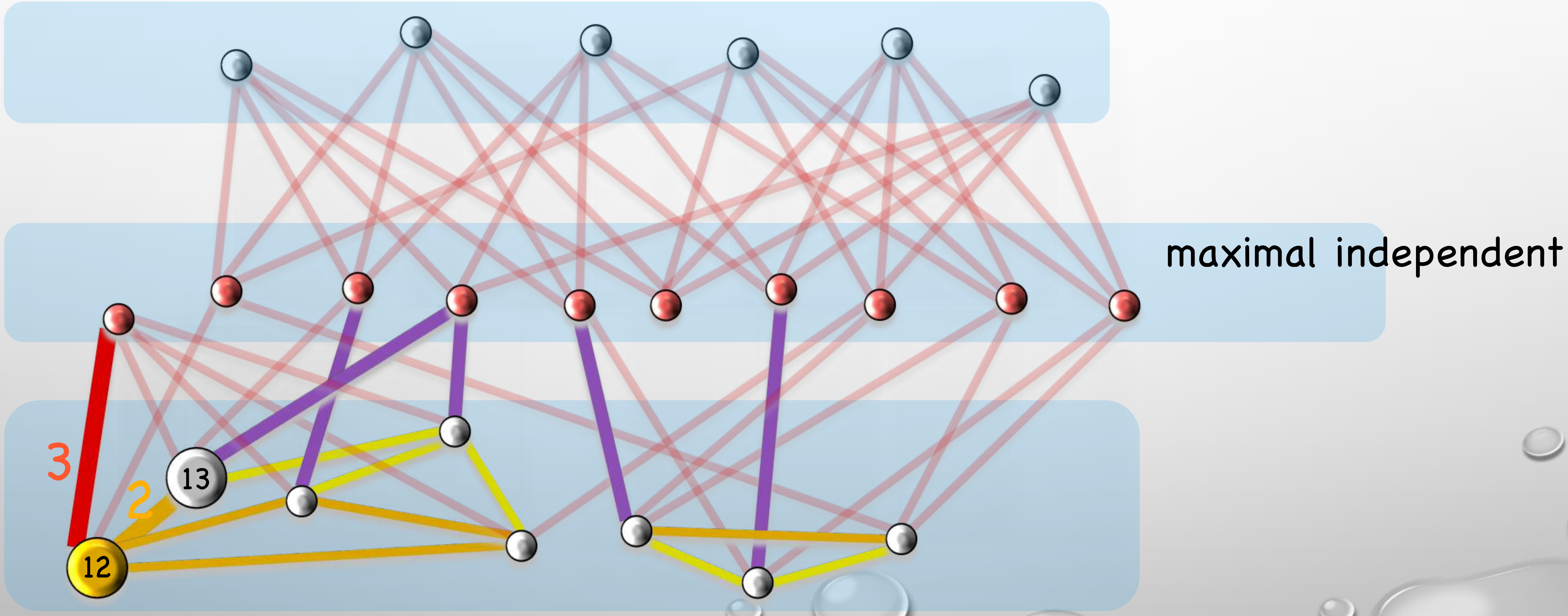

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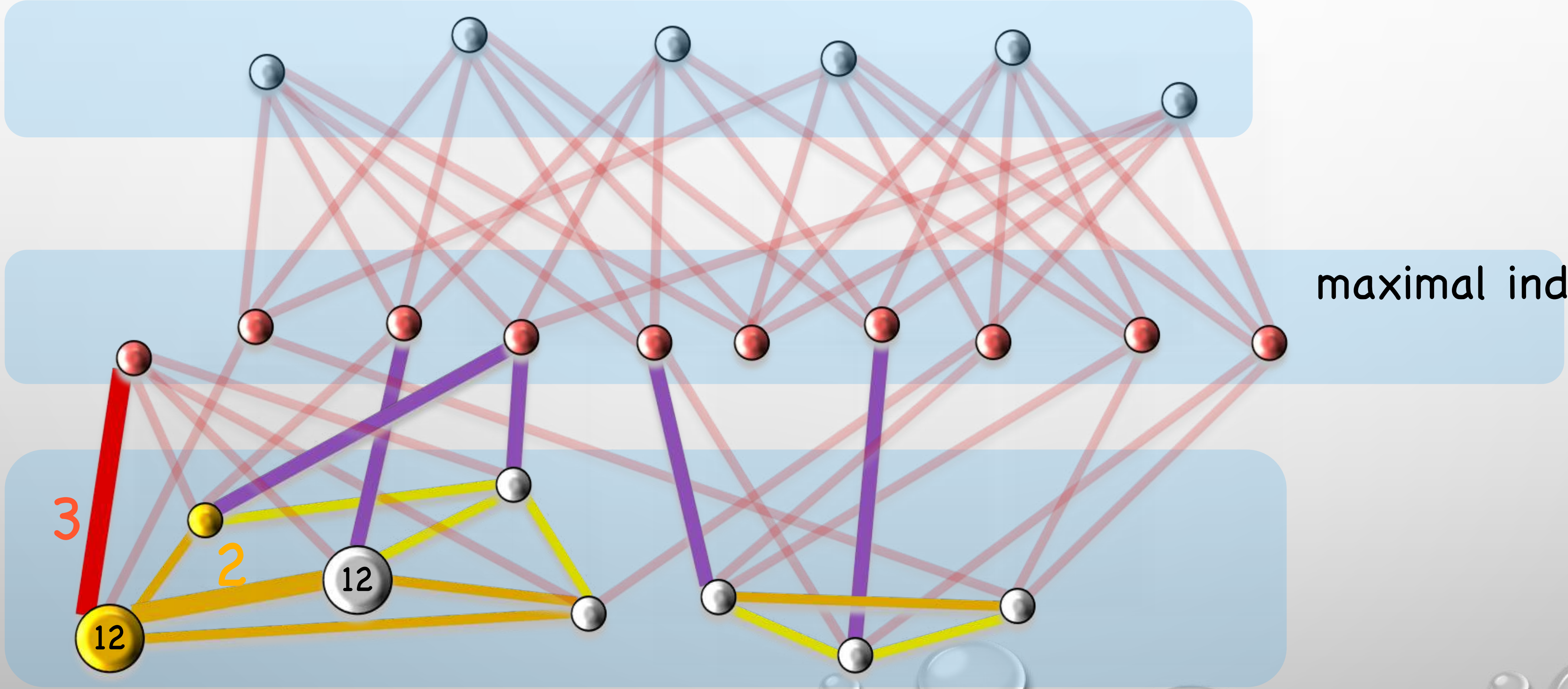

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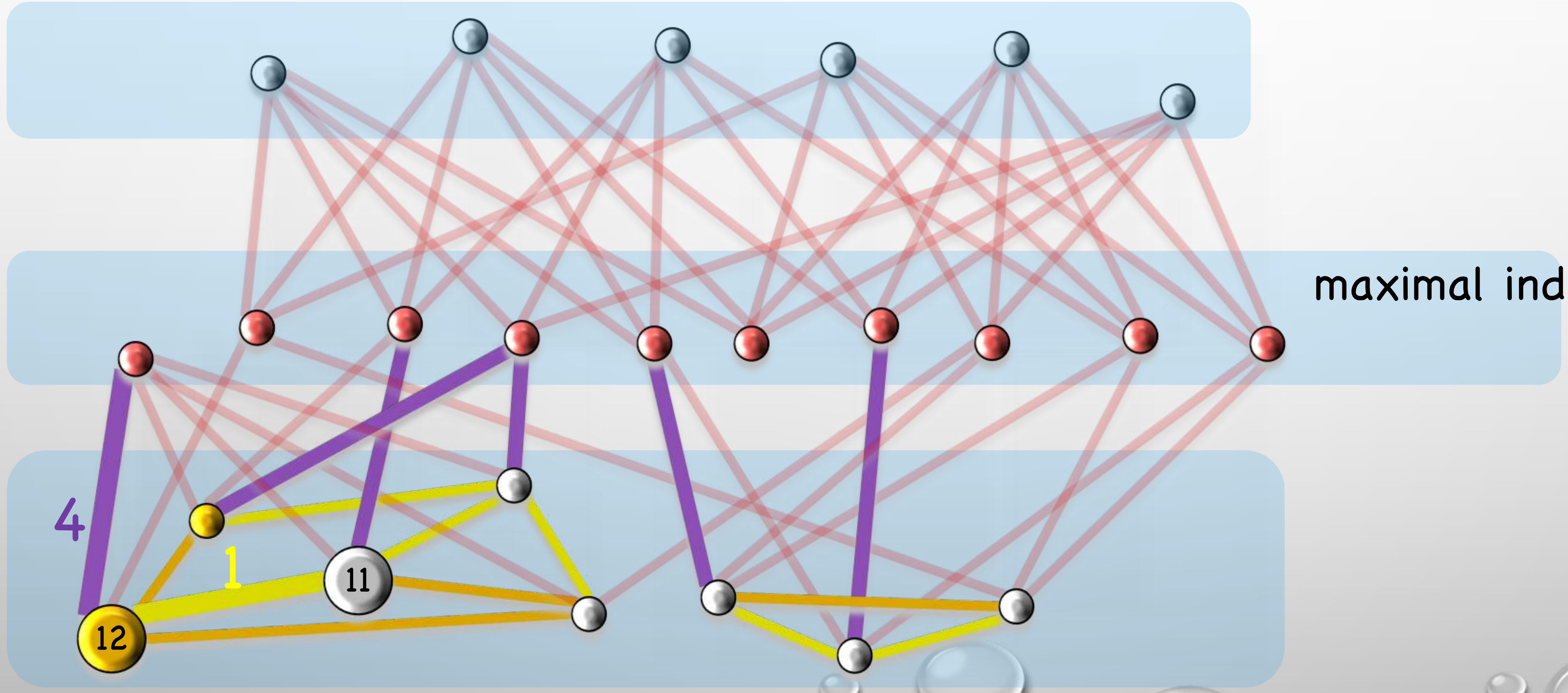

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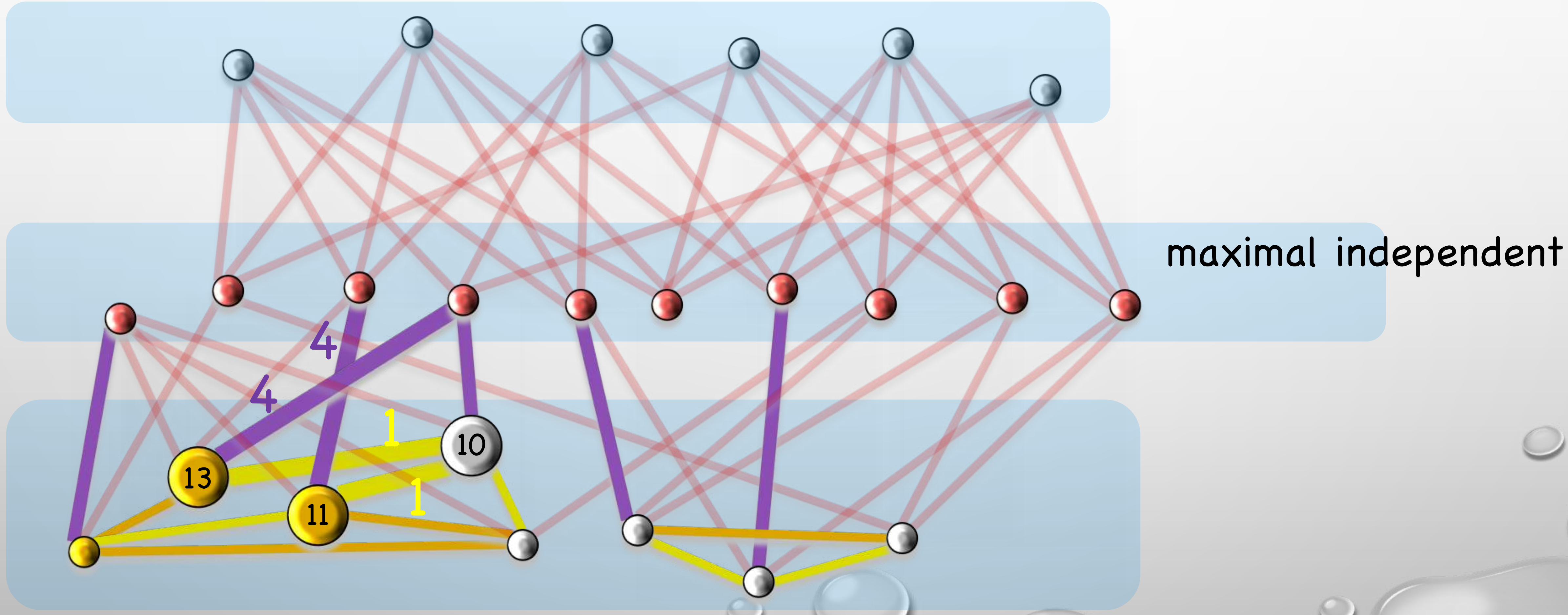

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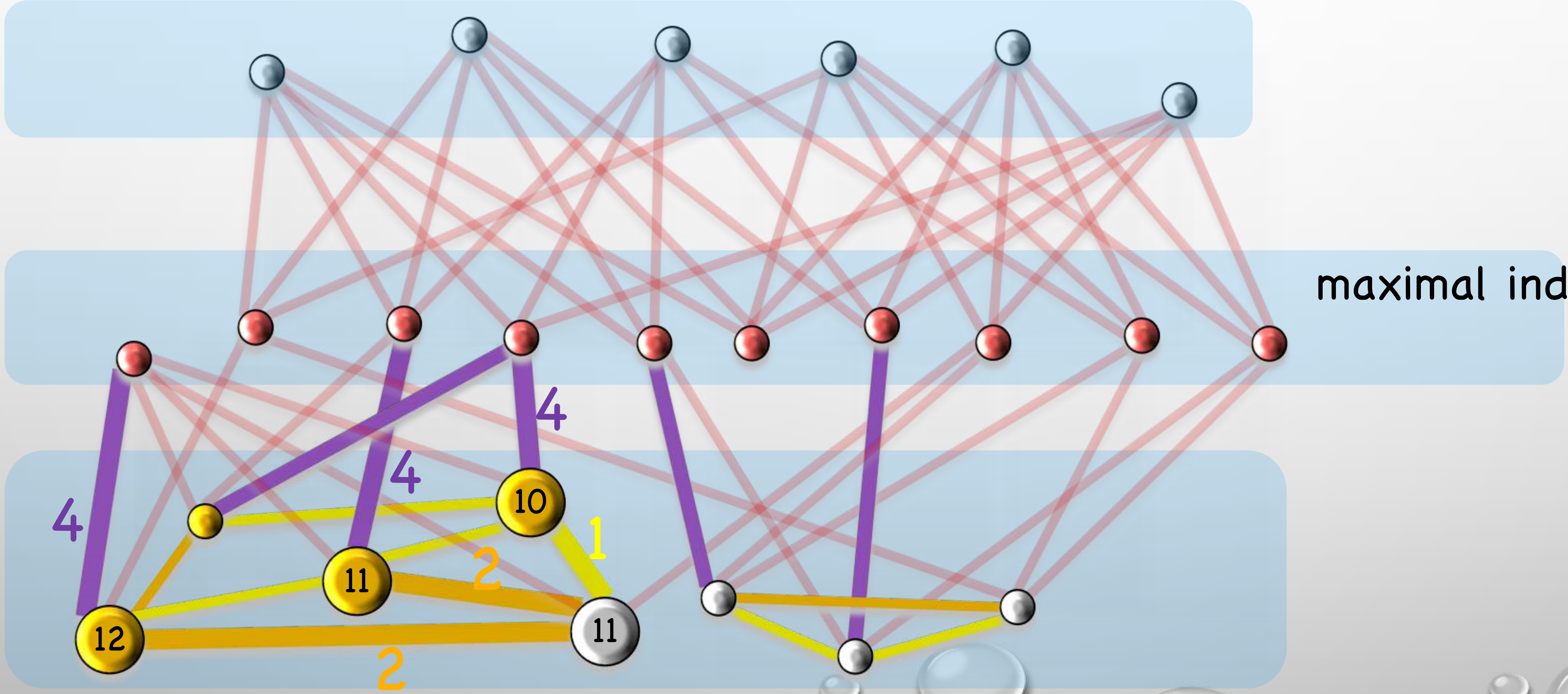

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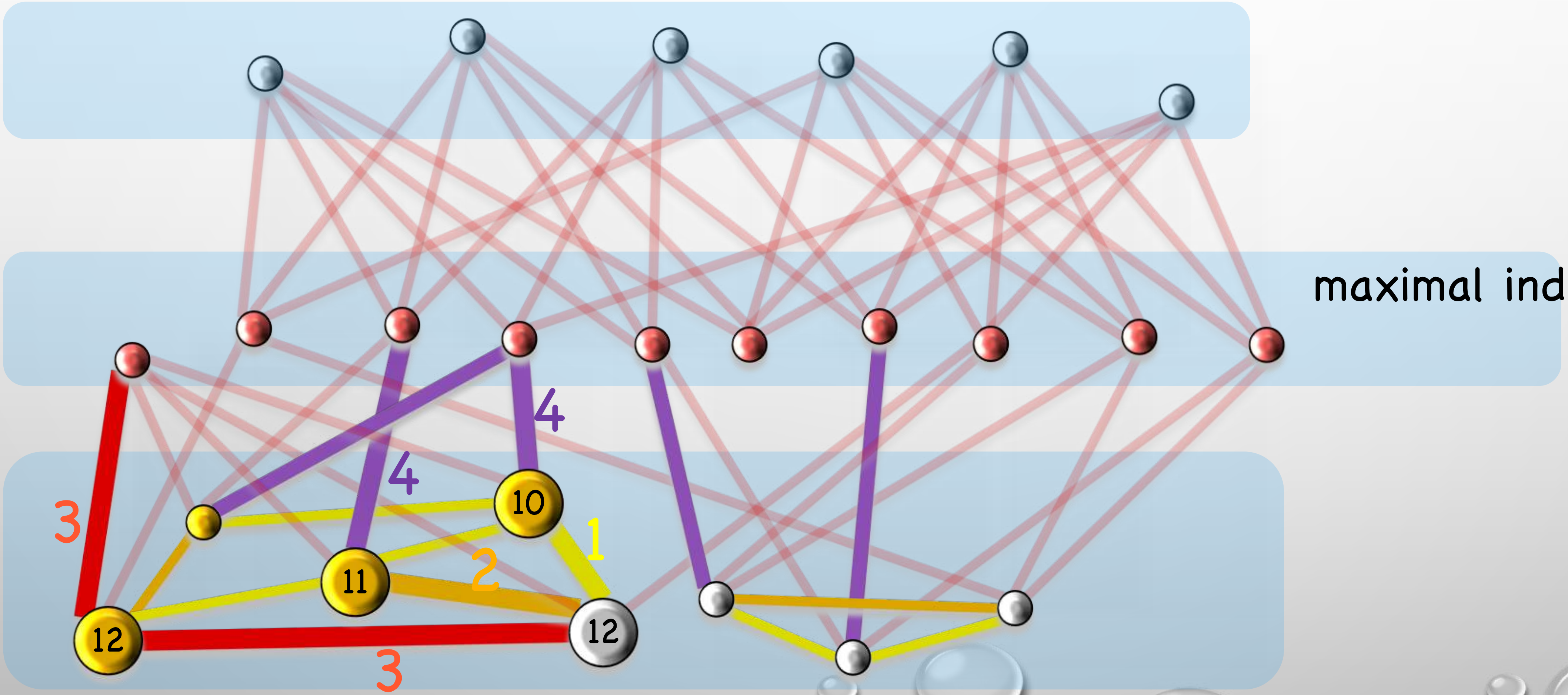

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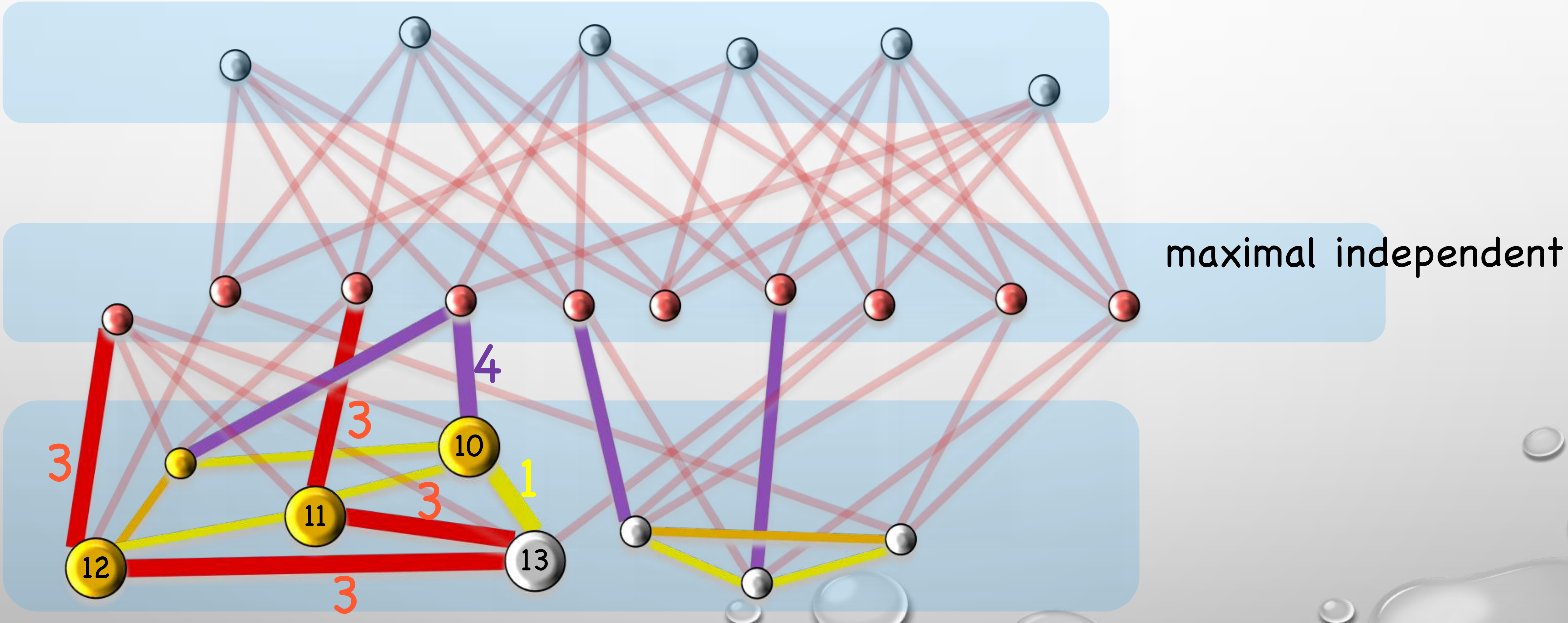

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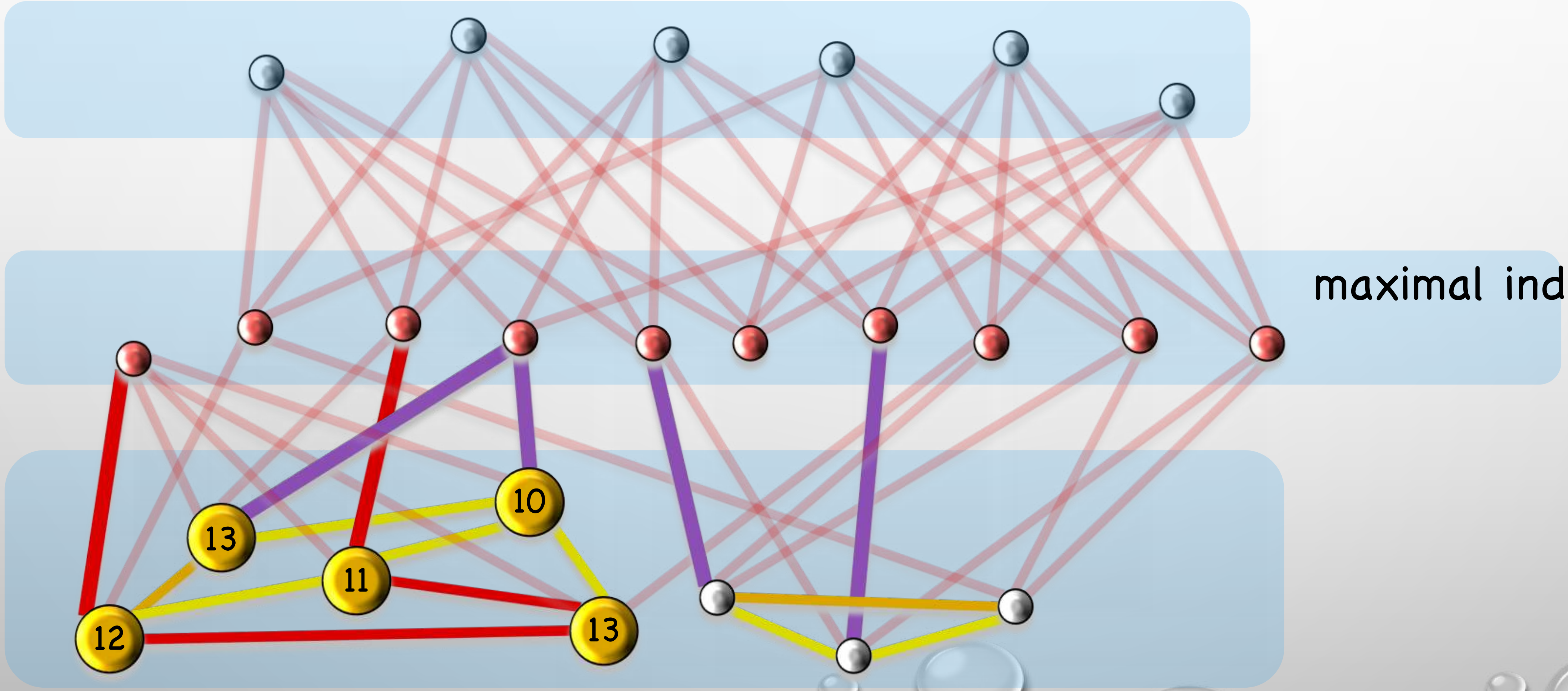

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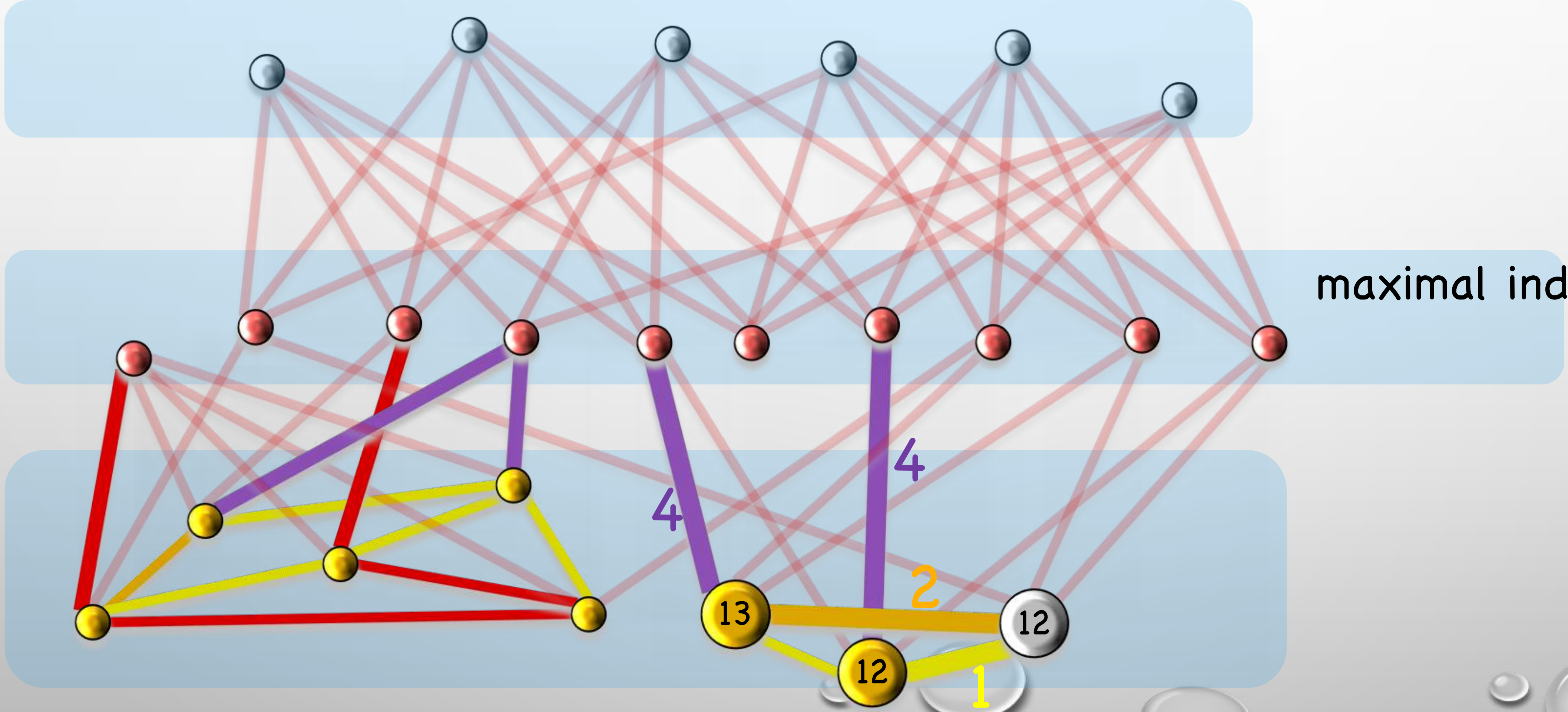

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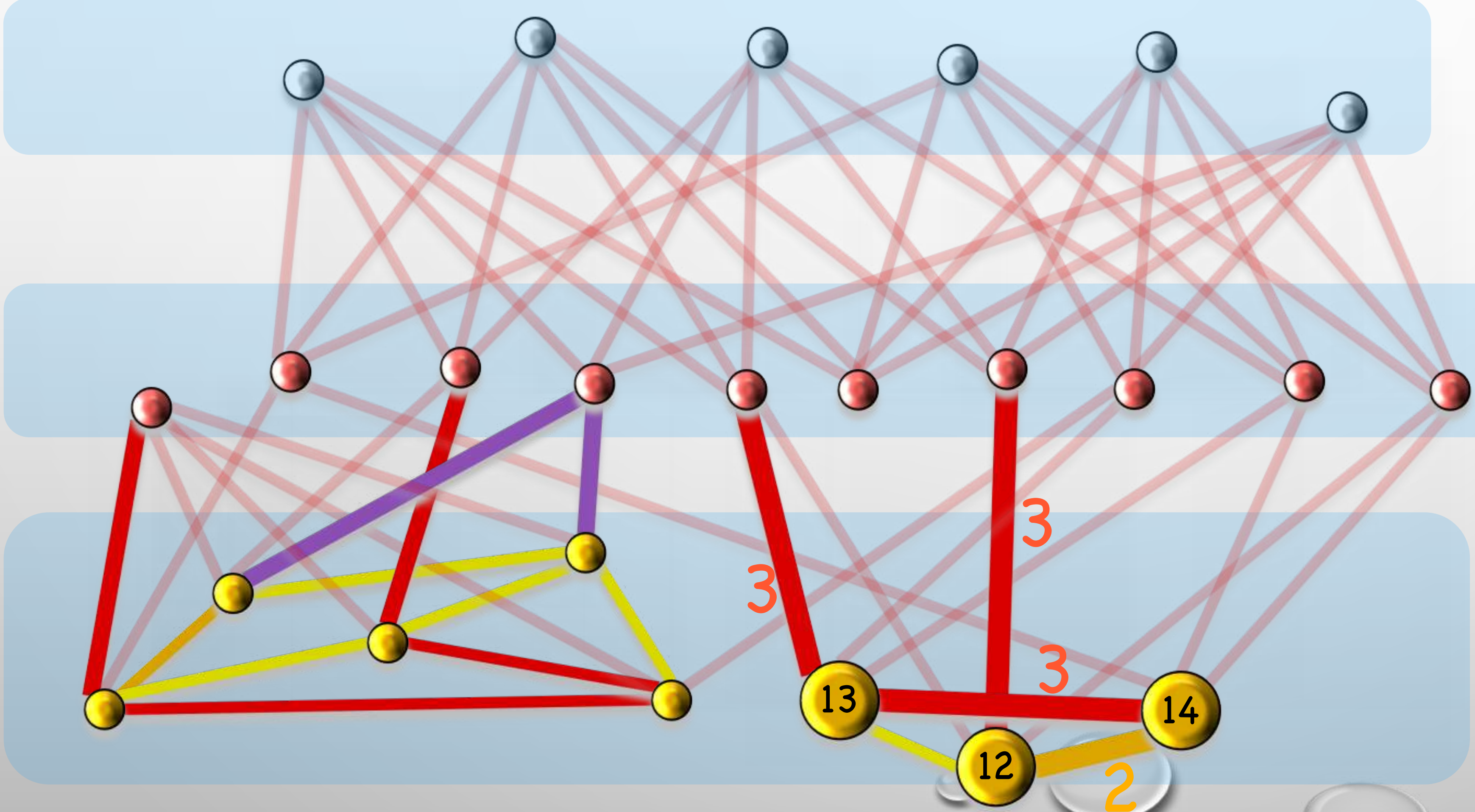

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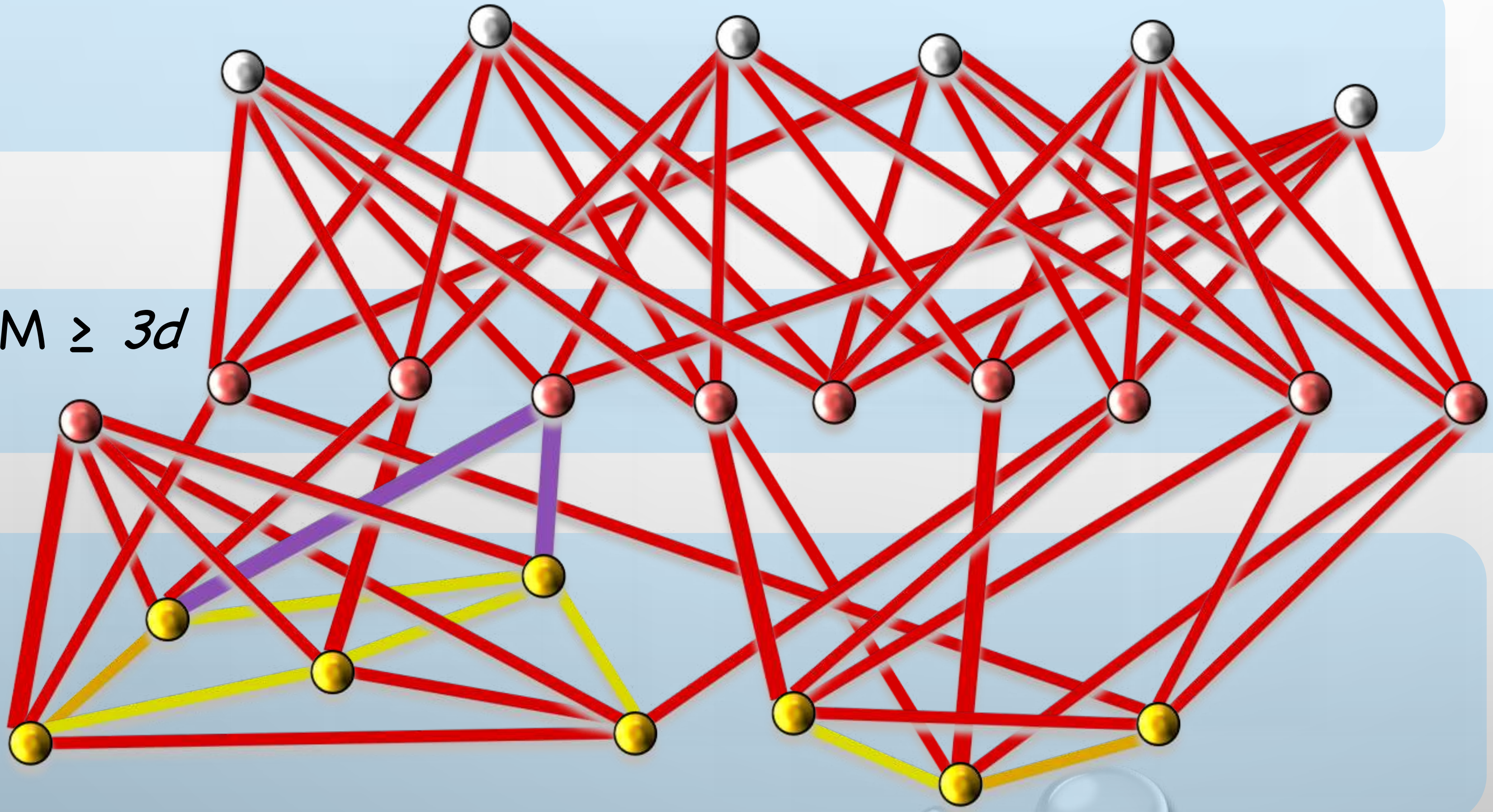
SUM ≥ 3*d*



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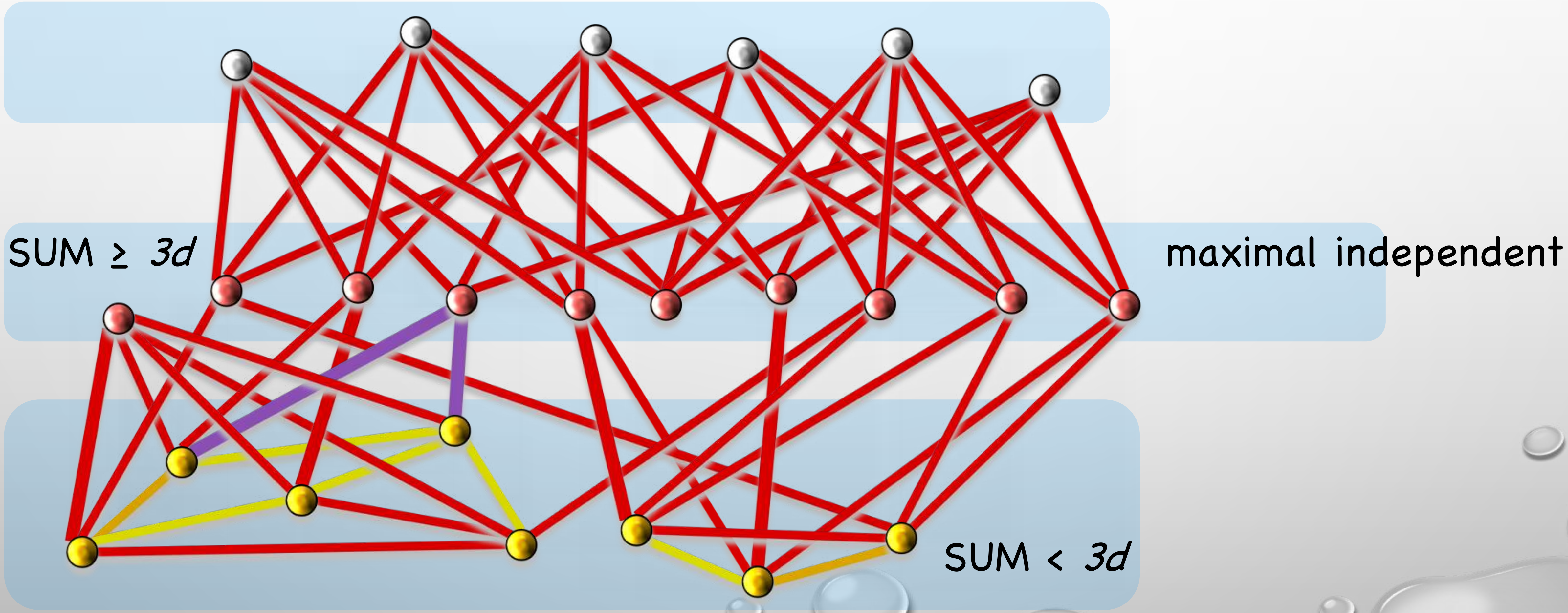

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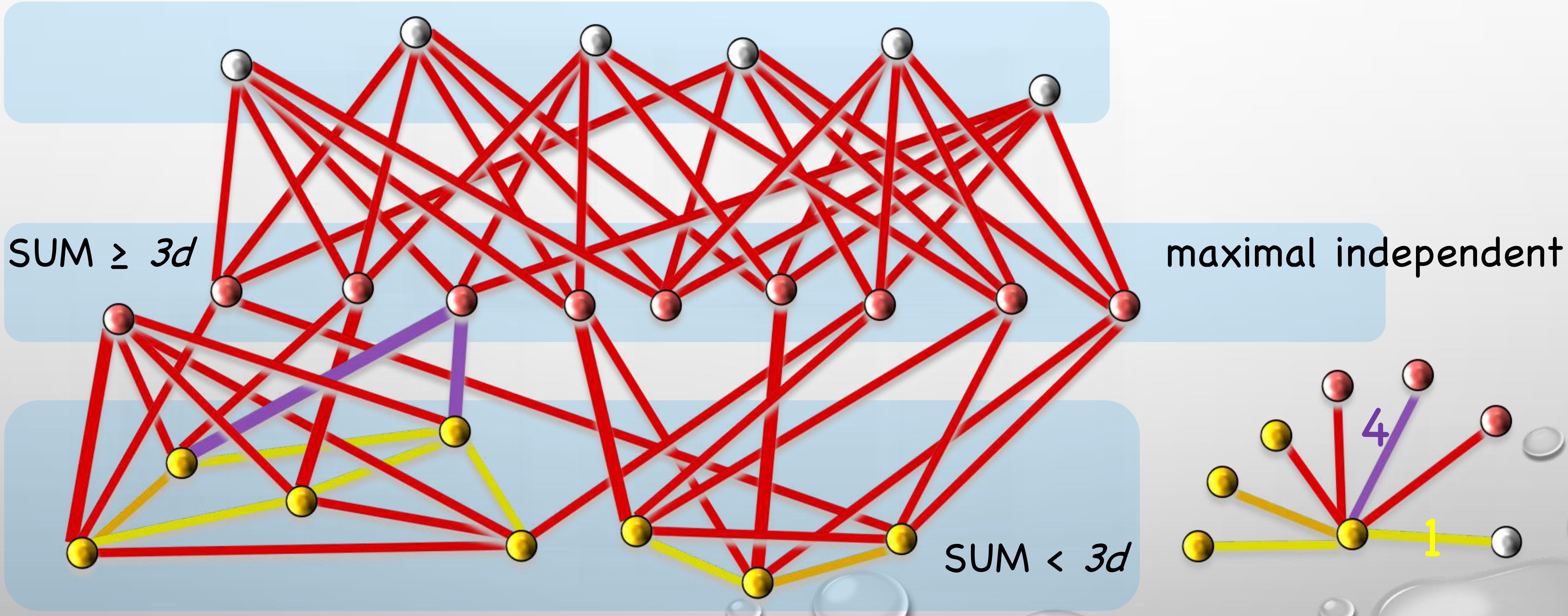

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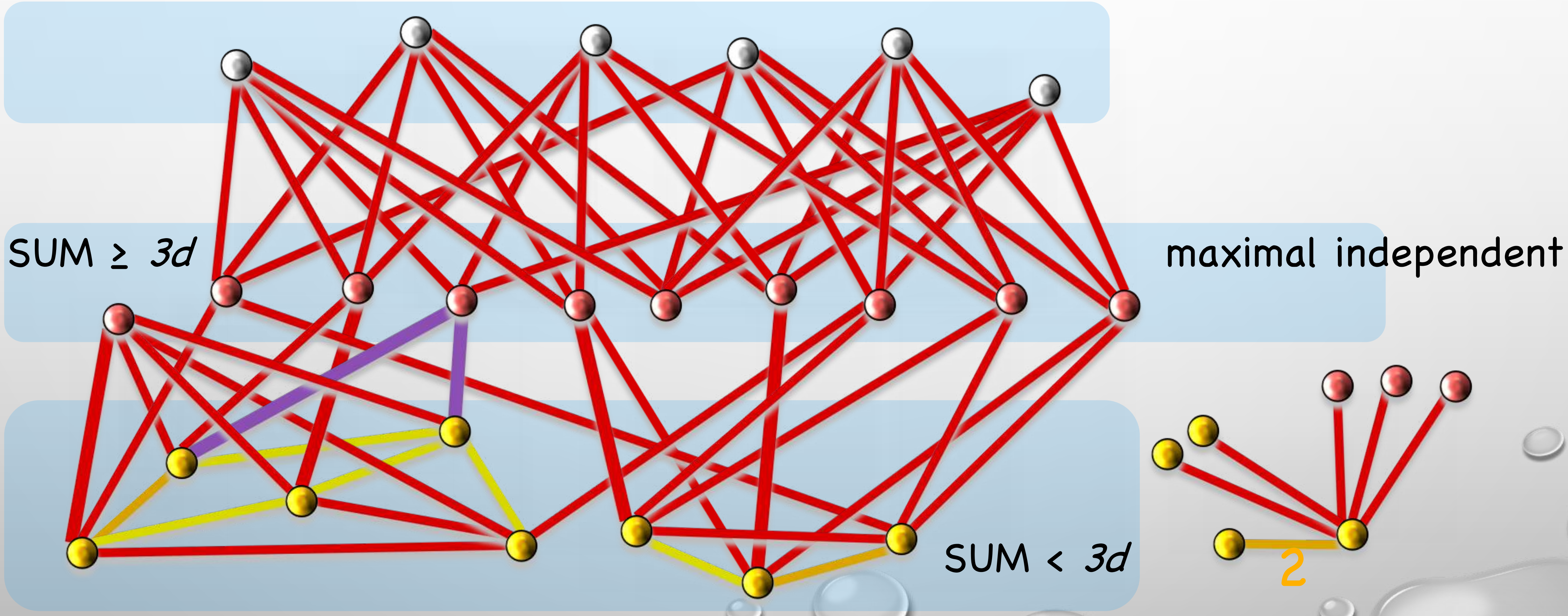

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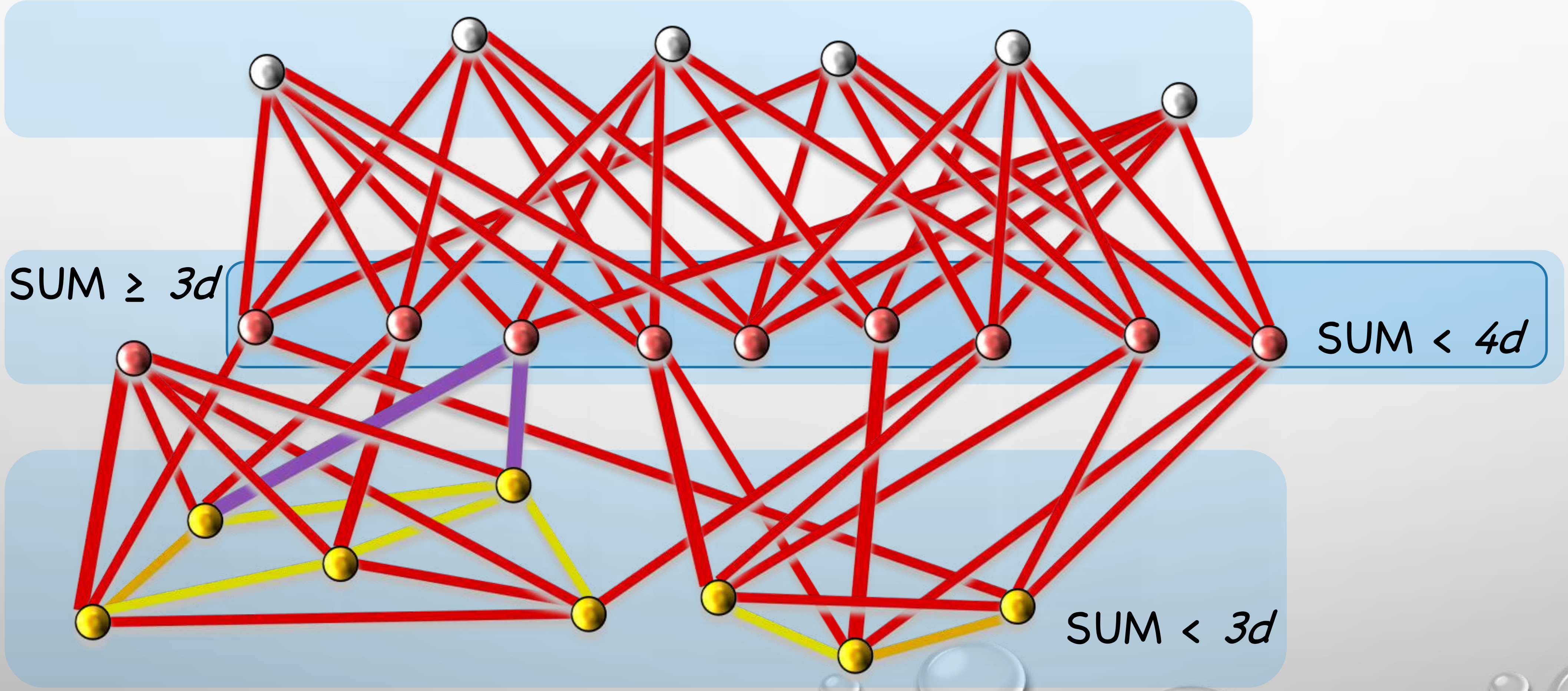
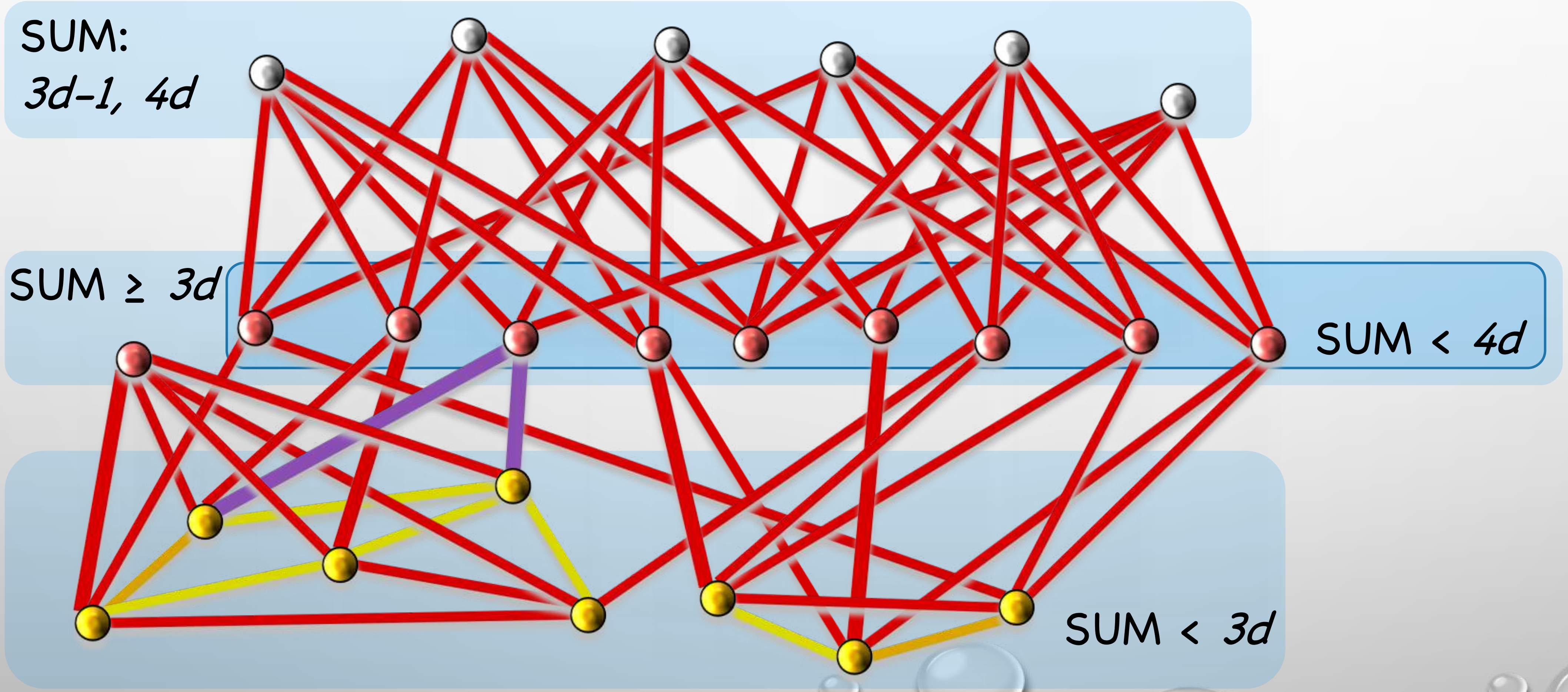
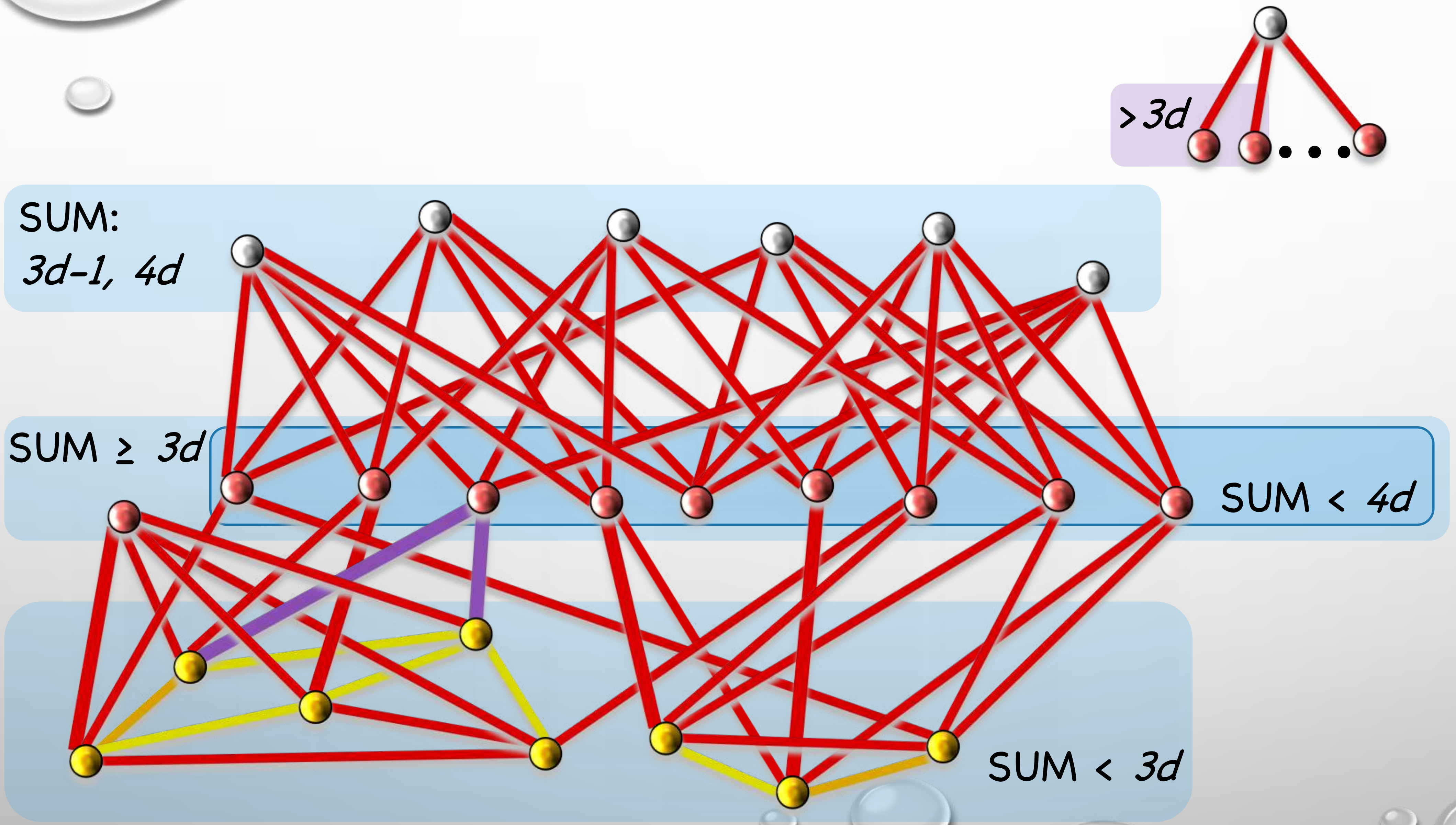


1-2-3-4-colouring of *d*-regular graphs



1-2-3-4-colouring of d-regular graphs



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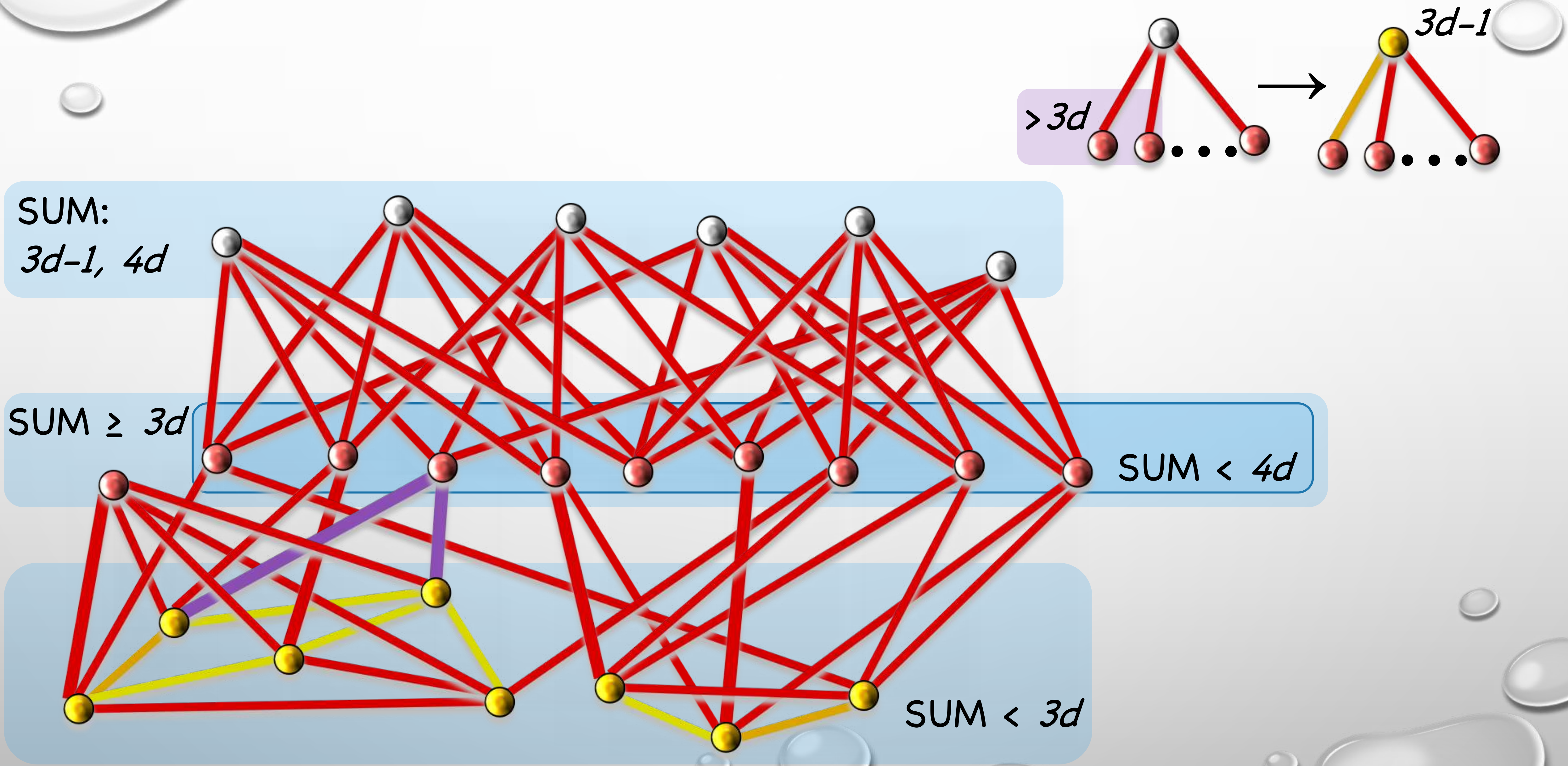

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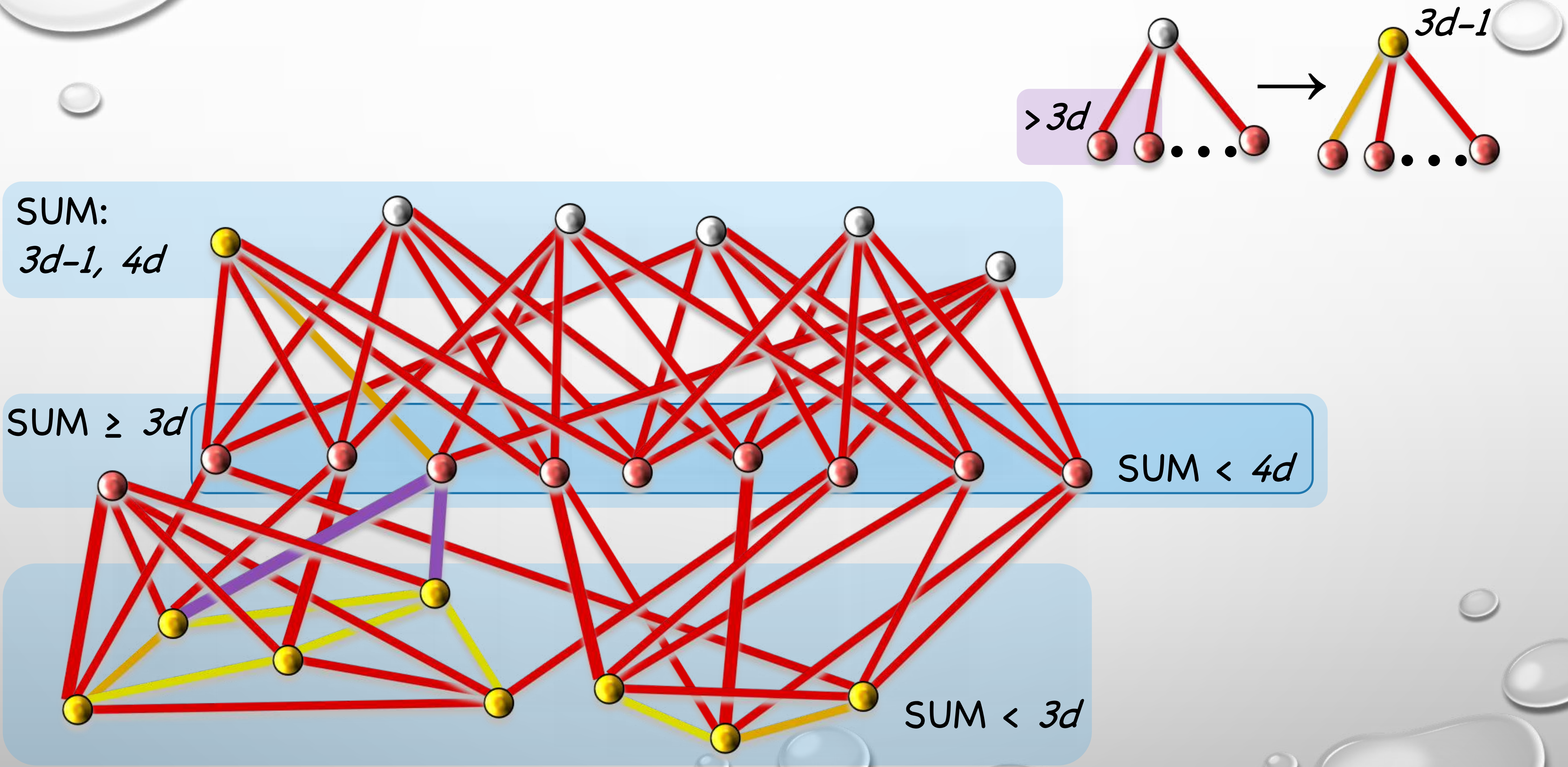

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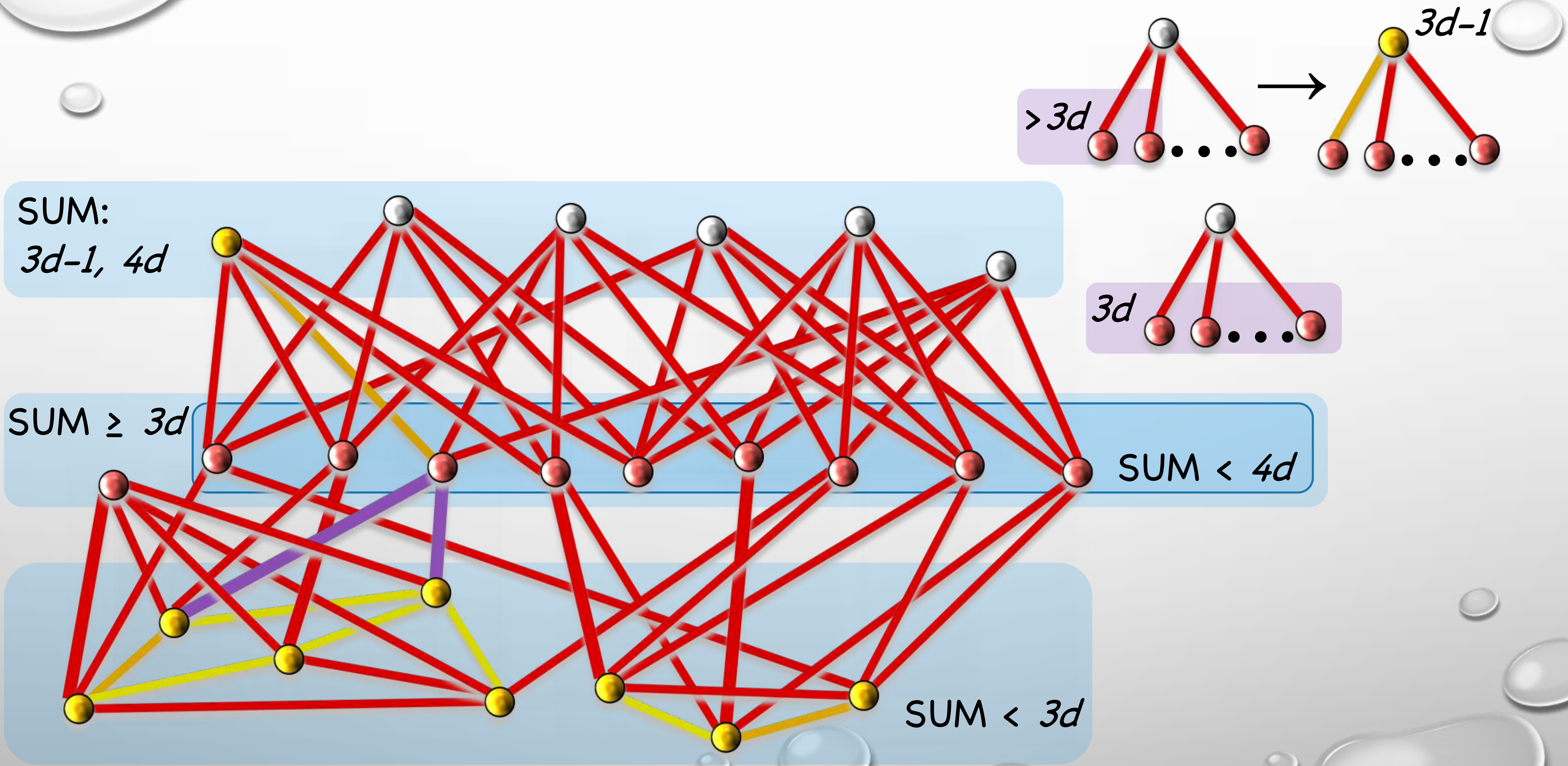

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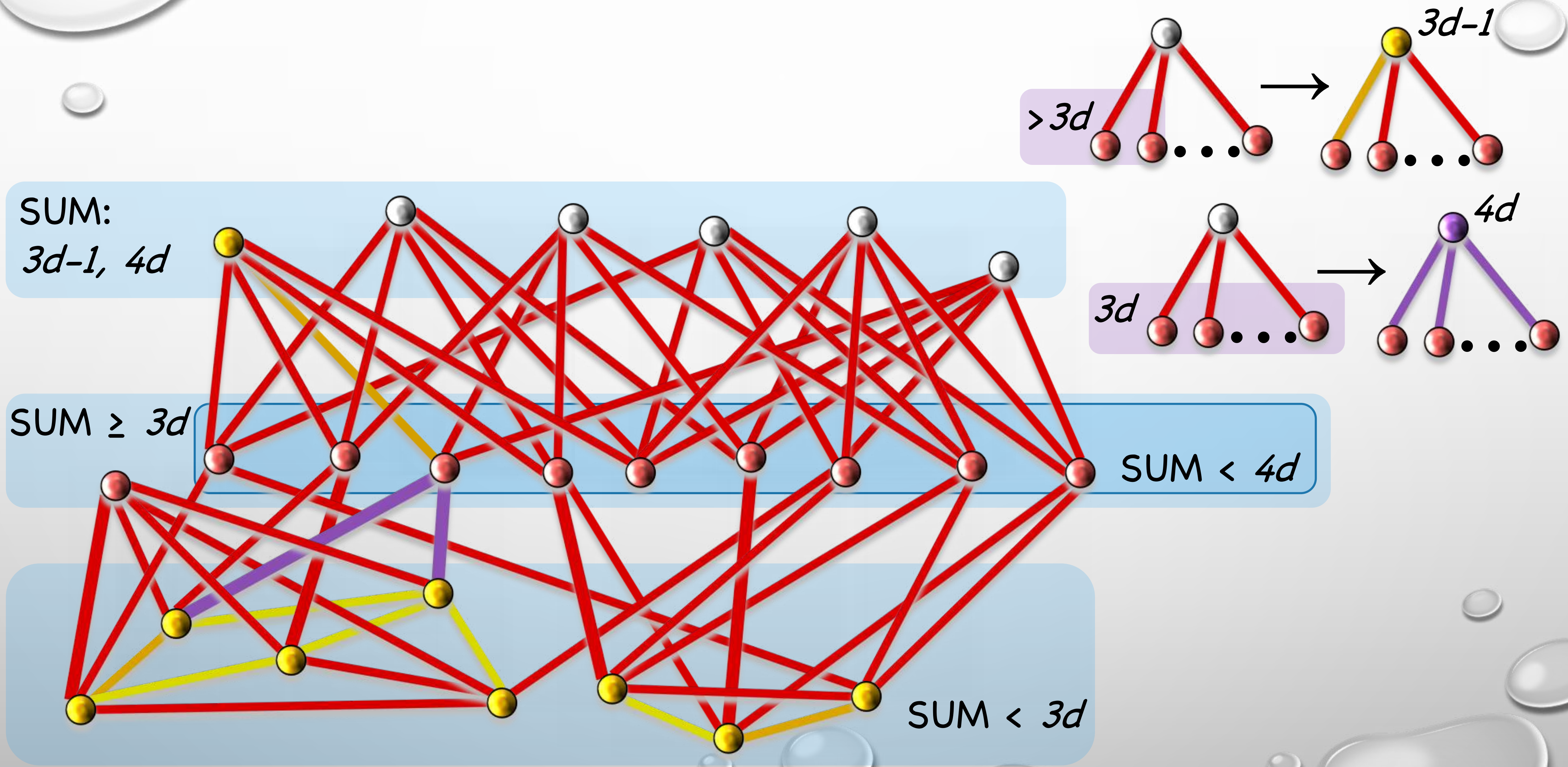

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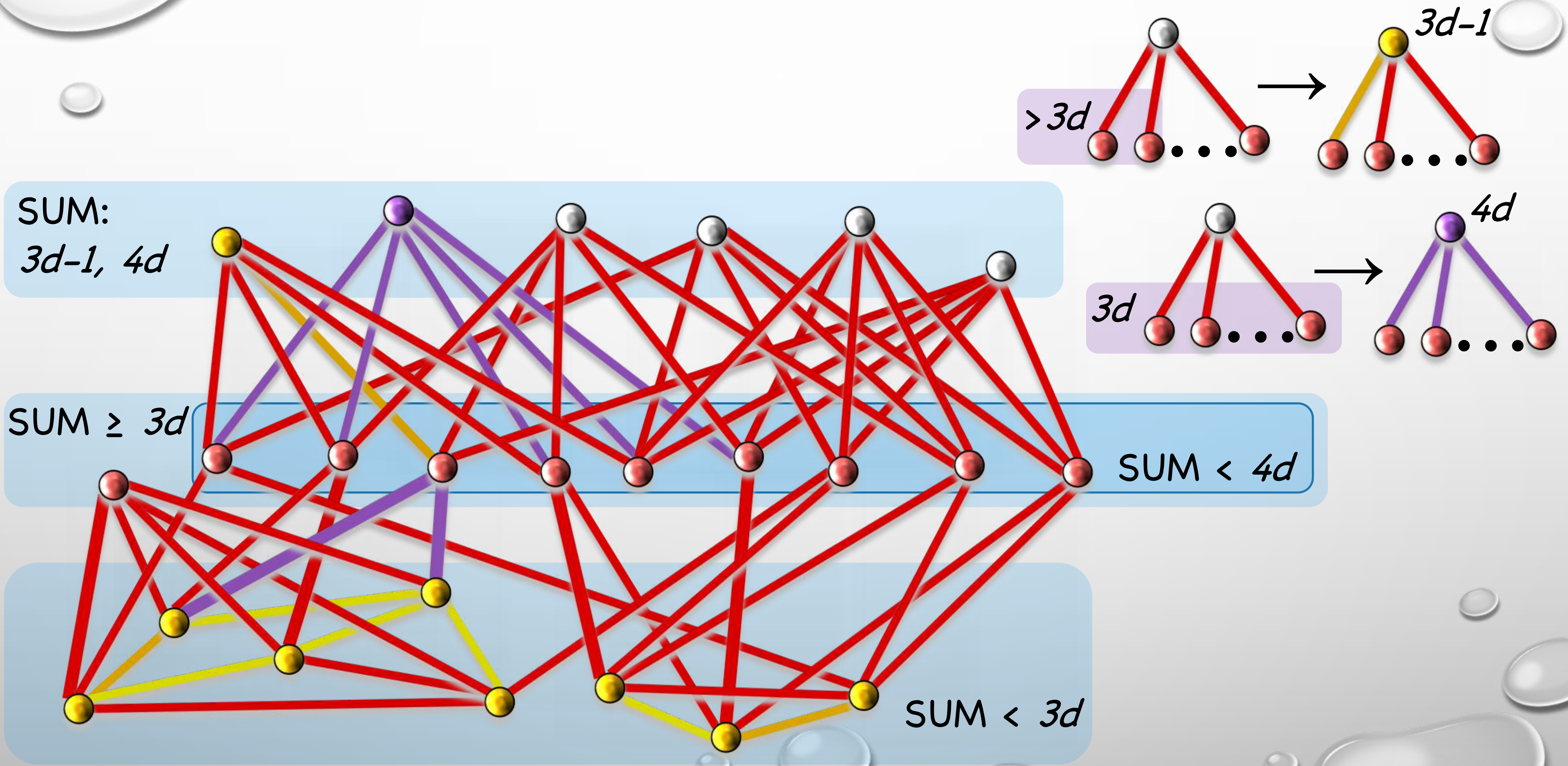

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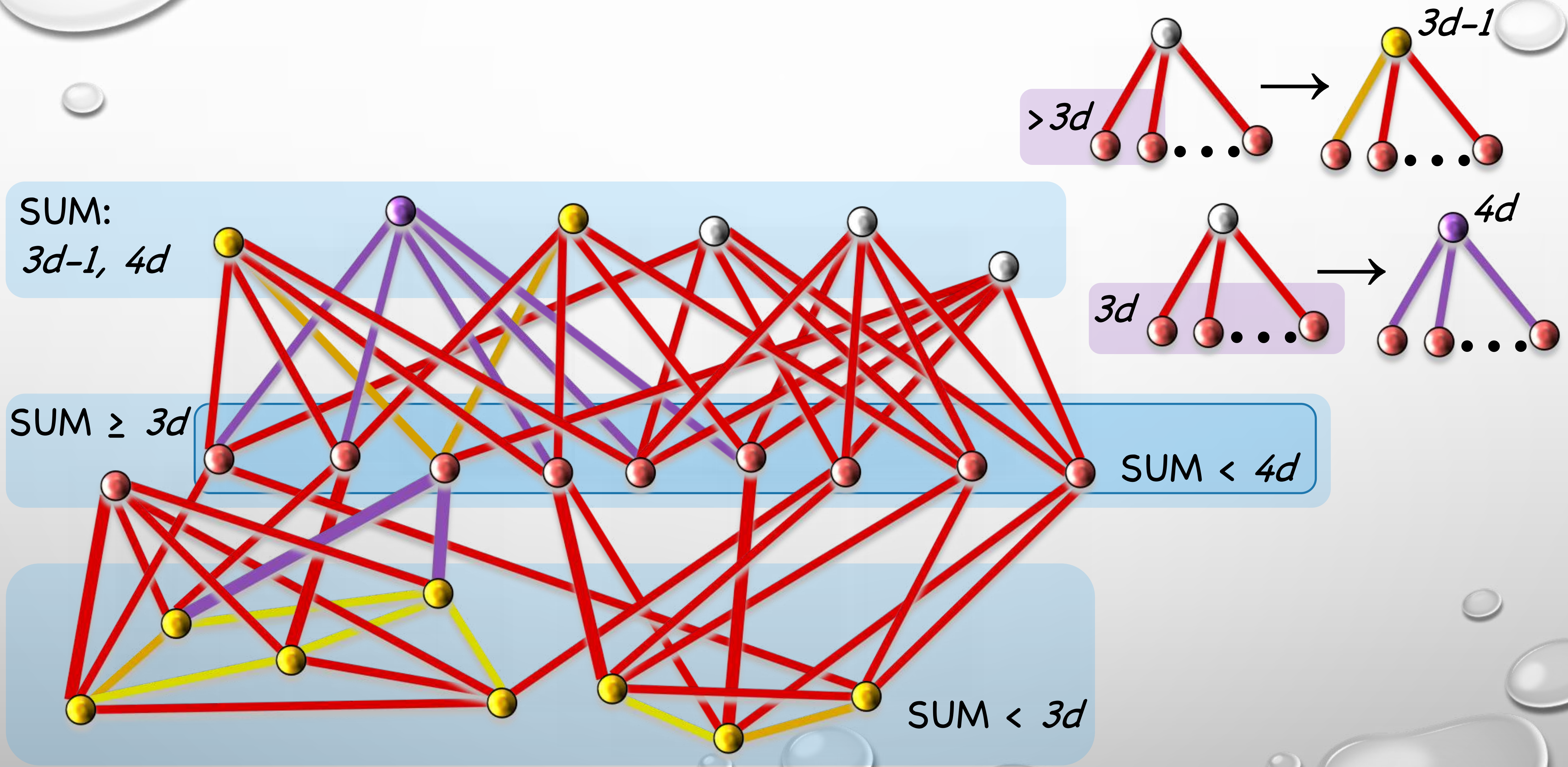

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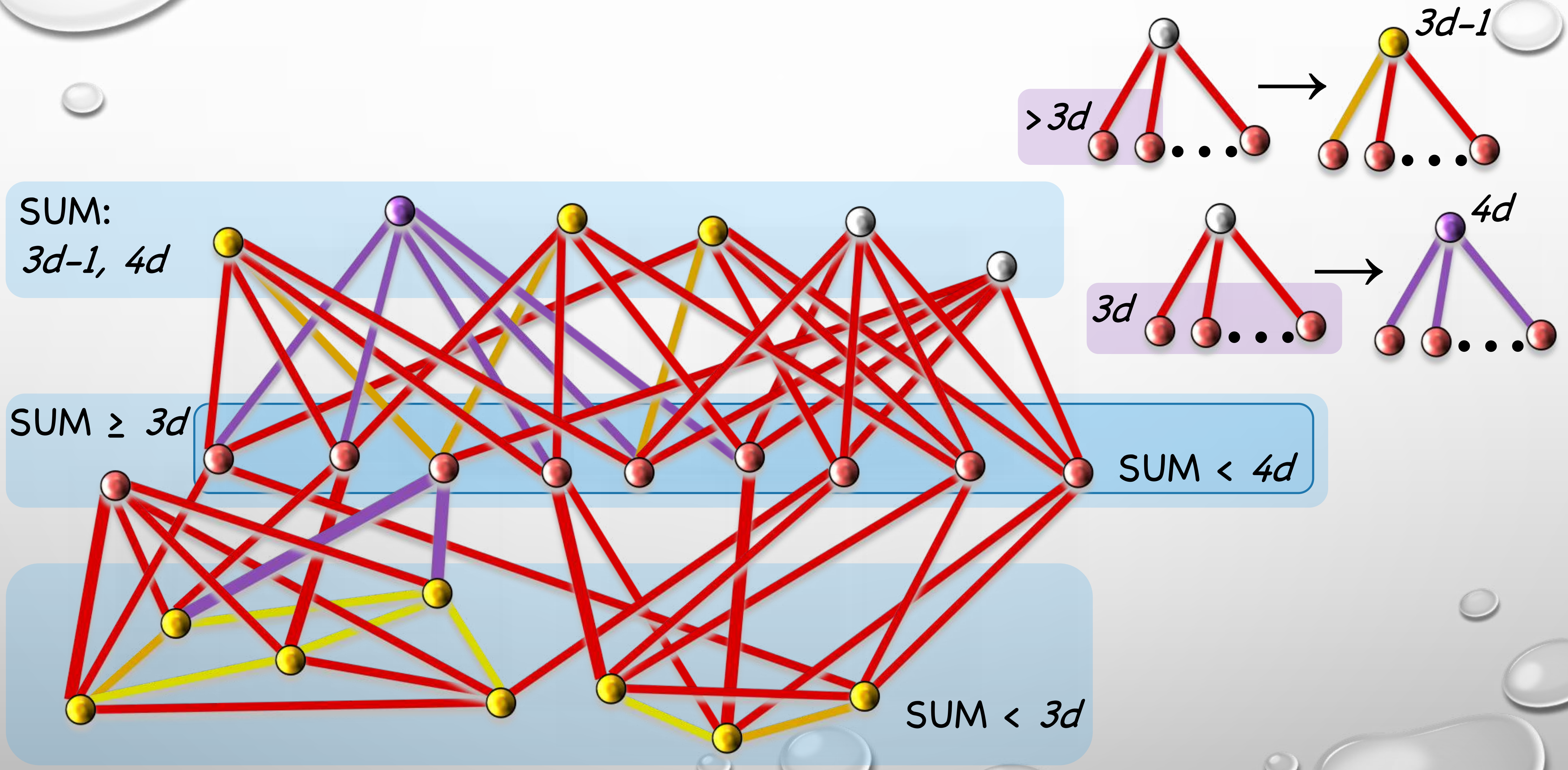

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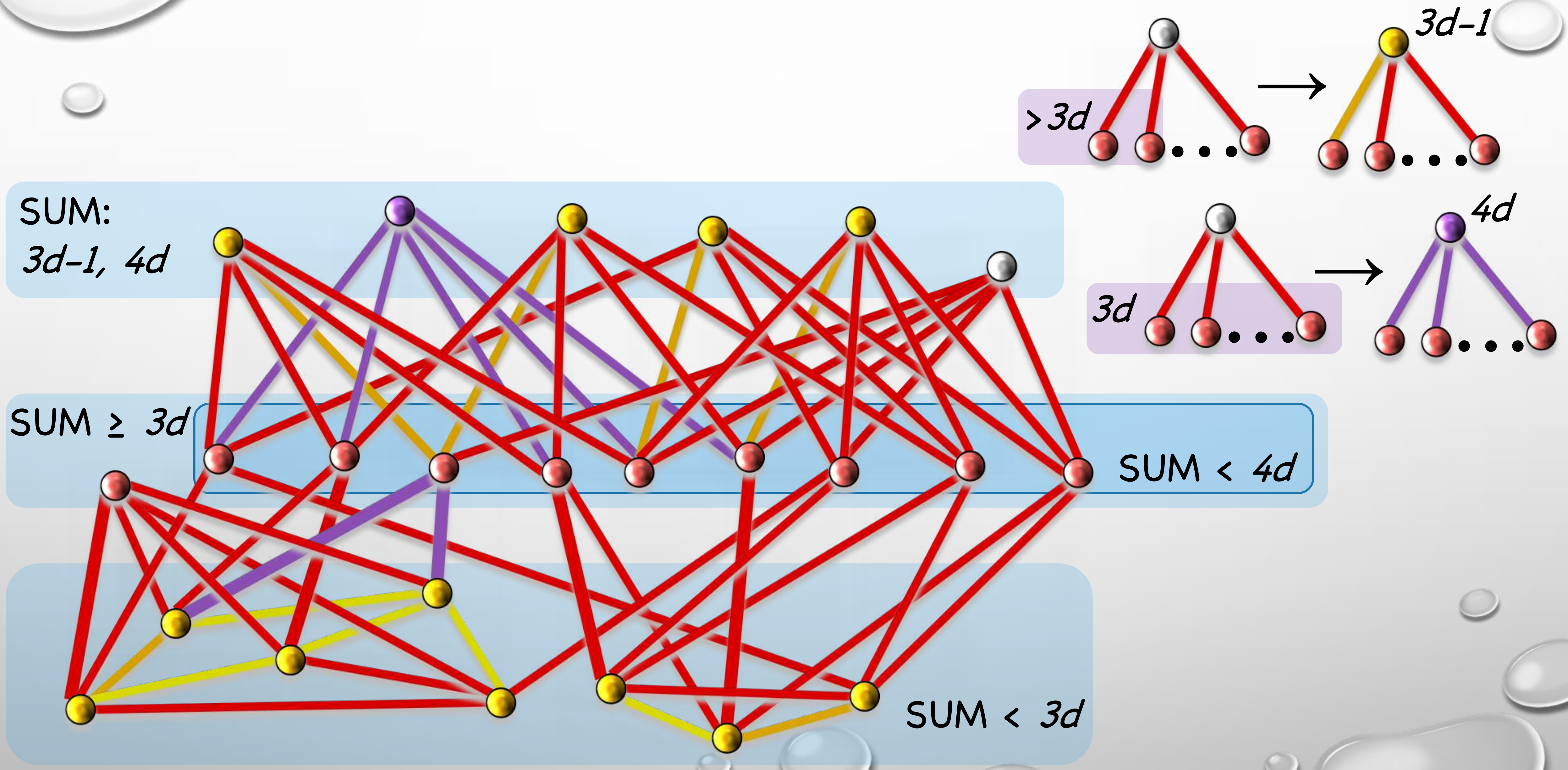

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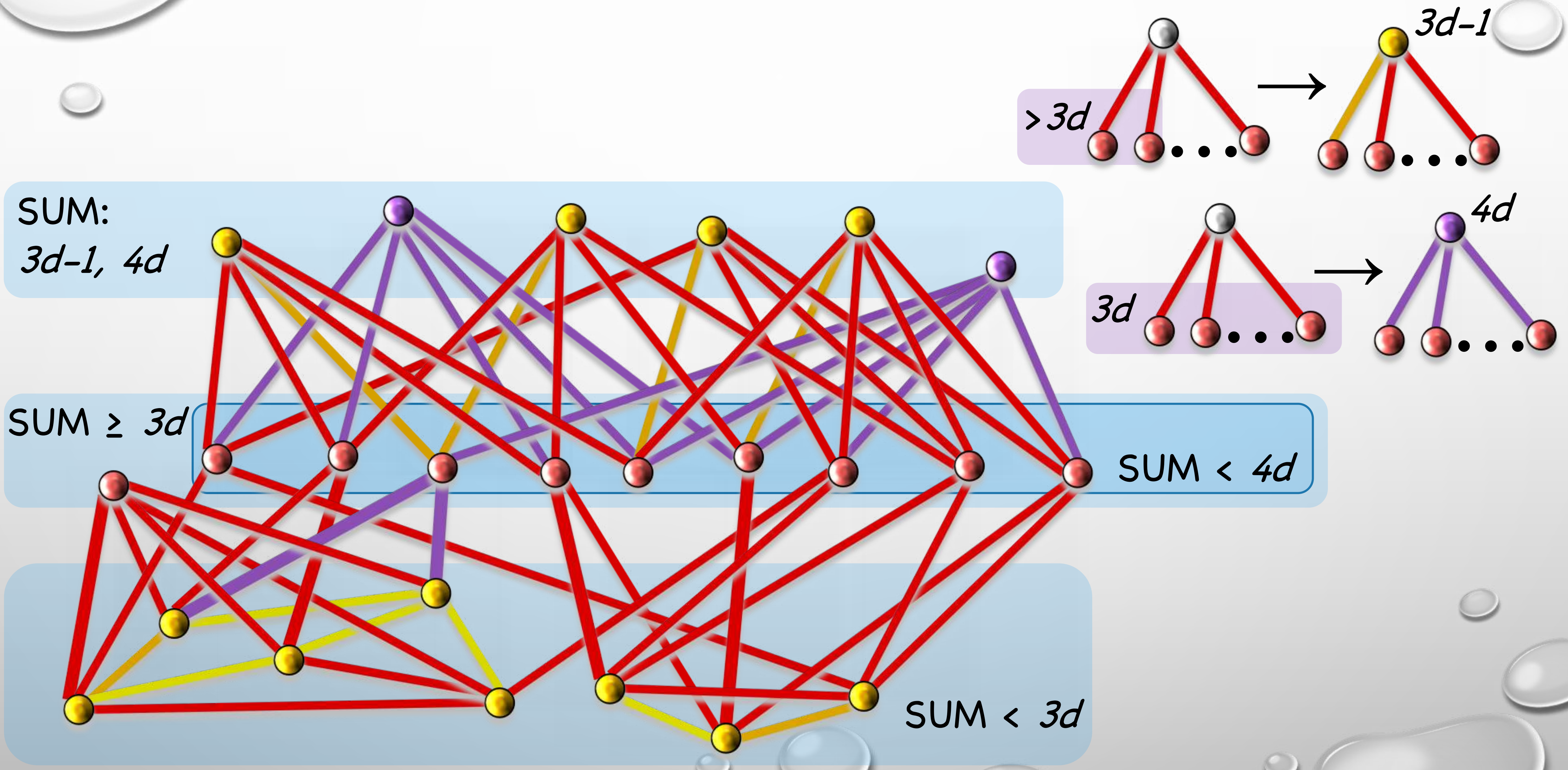

THE 1-2-3 CONJECTURE ALMOST ALMOST HOLDS FOR REGULAR GRAPHS

Th. Weights 1,2,3,4 suffice for regular graphs. (P. 2019+)

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Th. Weights 1,2,3 suffice for *d*-regular graphs with *d* large enough.
(P. 2019+)

Th. *Every graph is (2,3)-choosable* (Wong, Zhu 2016)

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Con. *Every graph is (2,2)-choosable* (Wong, Zhu 2011; P. Woźniak 2011)

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Con. *Every graph is (1,3)-choosable* (Wong, Zhu 2011)

Th. *Every graph is (2,3)-choosable* (Wong, Zhu 2016)

Con. *Every graph is (2,2)-choosable* (Wong, Zhu 2011; P. Woźniak 2011)
k

Con. *Every graph is (1,3)-choosable* (Wong, Zhu 2011)
k

THANK YOU!